The following is a review of the Derivative Investments principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

Forward Markets and Contracts

Study Session 16

Exam Focus

This topic review covers the calculation of price and value for forward contracts, specifically equity forward contracts, T-bond forward contracts, currency forwards, and forward (interest) rate agreements. You need to have a good understanding of the no-arbitrage principal that underlies these calculations, because it is used in the topic reviews of futures and swaps pricing as well. There are several important price and value formulas in this review. A clear understanding of the sources and timing of forward contract settlement payments will enable you to be successful on this portion of the exam without depending on pure memorization of these complex formulas. In the past, candidates have been tested on their understanding of the relationship of the payments at settlement to interest rate changes, asset price changes, and index level changes. The pricing conventions for the underlying assets have been tested separately. The basic contract mechanics are certainly "fair game," so don't overlook the easy stuff by spending too much time trying to memorize the formulas.

Warm-Up: Forward Contracts

The party to the forward contract that agrees to buy the financial or physical asset has a **long forward position** and is called the **long**. The party to the forward contract that agrees to sell/deliver the asset has a **short forward position** and is called the **short**.

We will illustrate the basic forward contract mechanics through an example based on the purchase and sale of a Treasury bill. Note that while forward contracts on T-bills are usually quoted in terms of a discount percentage from face value, we use dollar prices here to make the example easy to follow.

Consider a contract under which Party A agrees to buy a $1,000 face value 90-day Treasury bill from Party B 30 days from now at a price of $990. Party A is the long and Party B is the short. Both parties have removed uncertainty about the price they will pay or receive for the T-bill at the future date. If 30 days from now T-bills are trading at $992, the short must deliver the T-bill to the long in exchange for a $990 payment. If T-bills are trading at $988 on the future date, the long must purchase the T-bill from the short for $990, the contract price.

Each party to a forward contract is exposed to **default risk**, the probability that the other party (the counterparty) will not perform as promised. Typically, no money changes hands at the inception of the contract, unlike futures contracts in which each party posts an initial deposit called the **margin** as a guarantee of performance.

At any point in time, including the settlement date, the party to the forward contract with the negative value will owe money to the other side. The other side of the contract will have a positive value of equal amount. Following this example, if the T-bill price is $992 at the (future) settlement date and the short does not deliver the T-bill for $990 as promised, the short has defaulted.

Professor's Note: For more details on the basics of forward contracts, please see the volumes from the online Schweser Library that you received free with your purchase of the 2007 Study Notes.

Warm-Up: Forward Contract Price Determination

The No-Arbitrage Principle

The price of a forward contract is *not* the price to purchase the contract, because the parties to a forward contract typically pay nothing to enter into the contract at its inception. Here, *price refers to the contract price of the underlying asset under the terms of the forward contract*. This price may be a U.S. dollar or euro price but it is often expressed as an interest rate or currency exchange rate. For T-bills, the price will be expressed as an annualized percentage discount from face value; for coupon bonds, it will usually be expressed as a yield to maturity; for the implicit loan in a forward rate agreement (FRA), it will be expressed as annualized London Interbank Offered Rate (LIBOR); and for a currency forward, it is expressed as an exchange rate between the two currencies involved. However it is expressed, this rate, yield, discount, or dollar amount is the forward price in the contract.

The price that we wish to determine is the forward price that makes the *values* of both the long and the short positions zero at contract initiation. We will use the *no-arbitrage principle*: there should not be a riskless profit to be gained by a combination of a forward contract position with positions in other assets. This principle assumes that (1) transactions costs are zero, (2) there are no restrictions on short sales or on the use of short sale proceeds, and (3) both borrowing and lending can be done in unlimited amounts at the risk-free rate of interest. This concept is so important, we'll express it in a formula:

forward price = price that would not permit profitable riskless arbitrage in frictionless markets

A Simple Version of the Cost-of-Carry Model

In order to explain the no-arbitrage condition as it applies to the determination of forward prices, we will first consider a forward contract on an asset that costs nothing to store and makes no payments to its owner over the life of the forward contract. A zero-coupon (pure discount) bond meets these criteria. Unlike gold or wheat, it has no storage costs; unlike stocks, there are no dividend payments to consider; and unlike coupon bonds, it makes no periodic interest payments.

The general form for the calculation of the forward contract price can be stated as follows:

$$FP = S_0 \times (1 + R_f)^T$$

or

$$S_0 = \frac{FP}{(1 + R_f)^T}$$

where:
FP = forward price
S_0 = spot price at inception of the contract $(t = 0)$
R_f = annual risk-free rate
T = forward contract term in years

Example: Calculating the no-arbitrage forward price

Consider a 3-month forward contract on a zero-coupon bond with a face value of $1,000 that is currently quoted at $500, and assume a risk-free annual interest rate of 6%. Determine the price of the forward contract under the no-arbitrage principle.

Answer:

$$T = 3/12 = 0.25$$

$$FP = S_0 \times (1 + R_f)^T = \$500 \times 1.06^{0.25} = \$507.34$$

Professor's Note: If not given in the problem, assume STRIP and FRA calculations use 360 days. For forward contracts on coupon bonds, equities, and currencies, use 365 days. If the maturity is given in months, for example, use 1/12 for one month and 2/12 for two months.

Now, let's explore in more detail why $507.34 is the no-arbitrage price of the forward contract.

If FP > $507.34: A short position in the futures contract requires the delivery of this bond 90 days from now. The arbitrage that we examine in this case amounts to borrowing $500 at the risk-free rate of 6%, buying the bond for $500, and simultaneously taking the short position in the forward contract on the zero-coupon bond so that we are obligated to deliver the bond at the expiration of the contract for the forward price. At the settlement date, we can satisfy our obligation under the terms of the forward contract by delivering the zero-coupon bond, regardless of its market value at that time. We will use the payment we receive at settlement from the forward contract (the forward contract price) to repay the $500 loan. The total amount to repay the loan, since the term of the loan is 90 days of a 360-day year, is:

$$\text{loan repayment} = \$500 \times (1.06)^{90/360} = \$507.34$$

If the forward contract price is greater than this, the payment we receive as the short position in the contract is greater than our loan payoff and we have earned an arbitrage profit. This is called **cash and carry arbitrage**.

If FP < $507.34: We reverse the arbitrage trades from the previous case and generate an arbitrage profit as follows. We sell the bond short for $500 and simultaneously take the long position in the forward contract, which obligates us to purchase the bond in 90 days at the forward price. We lend the $500 proceeds from the short sale at the 6% annual rate for three months. In this case, at the settlement date, we receive the loan proceeds of $507.34, accept delivery of the bond in return for a payment equal to the forward price, and close out our short position by delivering the bond we just purchased at the forward price. Again, if the forward price is less than $507.34, we will have a profit equal to the amount by which the loan proceeds exceed the forward contract price. This is called **reverse cash-and-carry arbitrage**.

We can now determine that the no-arbitrage forward price that yields a zero *value* for both the long and short positions in the forward contract at inception is the no-arbitrage price of $507.34.

Professor's Note: This long explanation has answered the question, "What is the forward price that allows no arbitrage?" You'll have to trust me, but a very clear understanding here will make what follows easier and will serve you well as we progress to futures, options, and swaps.

Forward Contract Value

LOS 64.a: Explain how the value of a forward contract is determined at initiation, during the life of the contract, and at expiration.

If we denote the value of the long position in a forward contract at time *t* as V_t, the value of the long position at contract initiation, t = 0, is:

$$V_0\left(\text{of long position at initiation}\right) = S_0 - \left[\frac{FP}{\left(1+R_f\right)^T}\right]$$

Note that the no-arbitrage relation we derived in the prior section insures that the value of the long position (and of the short position) is zero.

$$\text{If } S_0 = \frac{FP}{\left(1+R_f\right)^T}\text{, then } V_0 = 0$$

The value of the long position in the forward contract during the life of the contract after *t* years (t < T) have passed (since the initiation of the contract) is:

$$V_t\left(\text{of long position during life of contract}\right) = S_t - \left[\frac{FP}{\left(1+R_f\right)^{T-t}}\right]$$

This is the same equation as above, but the spot price, S_t, will have changed, and the period for discounting is now the number of years remaining until contract expiration (T – t). This is a zero-sum game, so the value of the contract to the short position is the negative of the long position value:

$$V_t\left(\text{of short position during life of contract}\right) = \left[\frac{FP}{\left(1+R_f\right)^{T-t}}\right] - S_t$$

$$= -V_t\left(\text{of long position during life of contract}\right)$$

Notice that the forward price, *FP*, is the forward price agreed to at the initiation of the contract, not the current market forward price. In other words, as the spot and forward market prices change over the life of the contract, one side (i.e., short or long position) wins and the other side loses. For example, if the market spot and forward prices increase after the contract is initiated, the long position makes money, the value of the long position is positive, and the value of the short position is negative. If the spot and forward prices decrease, the short position makes money.

Professor's Note: Unfortunately, you must be able to use the forward valuation formulas on the exam. If you're good at memorizing formulas, that prospect shouldn't scare you too much. However, if you don't like memorizing formulas, here's another way to remember how to value a forward contract. The long position will pay the forward price (FP) at maturity (time T) and receive the spot price (S_T). The value of the contract to the long position at maturity is what he will receive less what he will pay: S_T – FP. Prior to maturity (at time T), the value to the long is the present value of S_T (which is the spot price at time t of S_t) less the present value of the forward price: $S_t - \left[\frac{FP}{(1+R_f)^{T-t}}\right]$. So on the exam, think "long position is spot price minus present value of forward price."

Example: Determining value of a forward contract prior to expiration

In our 90-day zero-coupon bond contract example, we determined that the no-arbitrage forward price was \$507.34. Suppose that after 60 days the spot price on the zero-coupon bond is \$515 and the risk-free rate is still 6%. Calculate the value of the long and short positions in the forward contract.

Answer:

$$V_{60}\left(\text{of long position after 60 days}\right) = \$515 - \frac{\$507.34}{1.06^{30/360}} = \$515 - \$504.88 = \$10.12$$

$$V_{60}\left(\text{of short position after 60 days}\right) = -\$10.12$$

Another way to see this is to note that because the spot price has increased to \$515, the current no-arbitrage forward price is:

$$FP = \$515 \times 1.06^{30/360} = \$517.51$$

The long position has made money (and the short position has lost money) because the forward price has *increased* by \$10.17 from \$507.34 to \$517.51 since the contract was initiated. The value of the long position today is the present value of \$10.17 for 30 days at 6%:

$$V_{60}(\text{long position after 60 days}) = \frac{\$10.17}{1.06^{30/360}} = \$10.12$$

At contract expiration, we do not need to discount the forward price because the time left on the contract is zero. Since the long can buy the asset for FP and sell it for the market price S_T, the value of the long position is the amount the long position will receive if the contract is settled in cash:

$$V_T\left(\text{of long position at maturity}\right) = S_T - FP$$

$$V_T\left(\text{of short position at maturity}\right) = FP - S_T = -V_T\left(\text{of long position at maturity}\right)$$

Look for these ways in which the valuation of a forward contract might appear as part of an exam question:

- To mark-to-market for financial statement reporting purposes.
- To mark-to-market because it is required as part of the original agreement. For example, the two parties might have agreed to mark-to-market a 180-day forward contract after 90 days to reduce credit risk.
- To measure credit exposure (see LOS 64.e).
- To calculate how much it would cost to terminate the contract.

LOS 64.b: Distinguish an off-market forward contract from the more standard type of forward contract.

An **off-market forward contract** refers to one where there is a payment at initiation to either the long or the short because the forward price (for whatever reason) is not set to the no-arbitrage price. In other words, the value at contract initiation is not equal to zero, and either the long or the short must make a payment to the other at the initiation of the contract.

Suppose, for example, that the two parties negotiated the 90-day forward price for the $500 zero-coupon bond forward contract to be $510 instead of the no-arbitrage price of $507.34. The value of the long position at initiation would be:

$$V_0(\text{of long postion at initiation}) = \$500 - \left(\frac{\$510}{1.06^{90/360}}\right) = -\$2.62$$

In order to insure that the value of the contract is zero at initiation, the short position must pay the long position the $2.62 up front. If the contracted forward price is set below the no-arbitrage price, the long position pays the short position.

Equity Forward Contracts

LOS 64.c: Calculate and interpret the price and the value of an equity forward contract, given the different possible patterns of dividend payments.

Recall that the no-arbitrage forward price in our earlier example was calculated for an asset with no periodic payments. A stock, a stock portfolio, or an equity index may have expected dividend payments over the life of the contract. In order to price such a contract we must either adjust the spot price for the present value of the expected dividends (PVD) over the life of the contract or adjust the forward price for the future value of the dividends (FVD) over the life of the contract. The **no-arbitrage price of an equity forward contract** in either case is:

$$FP(\text{on an equity security}) = (S_0 - PVD) \times (1 + R_f)^T$$

$$FP(\text{on an equity security}) = \left[S_0 \times (1 + R_f)^T\right] - FVD$$

Professor's Note: In practice, we would calculate the present value from the ex-dividend date, not the payment date. On the exam, use payment dates unless the ex-dividend dates are given.

Example: Calculating the price of a forward contract on a stock

Calculate the no-arbitrage forward price for a 100-day forward on a stock that is currently priced at $30.00 and is expected to pay a dividend of $0.40 in 15 days, $0.40 in 85 days, and $0.50 in 175 days. The annual risk-free rate is 5% and the yield curve is flat.

Answer:

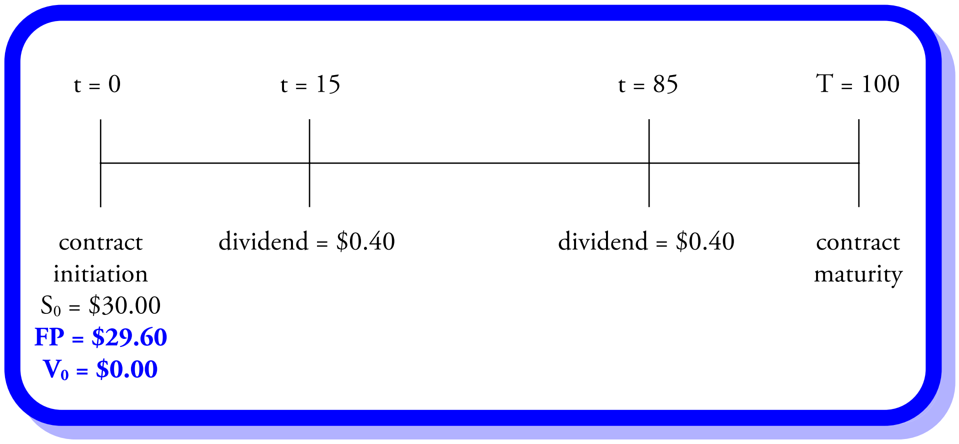
Ignore the dividend in 175 days because it occurs after the maturity of the forward contract. The time line of cash flows is shown in Figure 1.

$$PVD = \frac{\$0.40}{1.05^{15/365}} + \frac{\$0.40}{1.05^{85/365}} = \$0.7946$$

$$FP = (\$30.00 - \$0.7946) \times 1.05^{100/365} = \$29.60$$

Professor's Note: Remember, if not given in the problem, assume STRIP and FRA calculations use 360 days. For forward contracts on coupon bonds, equities, and currencies, use 365 days.

Figure 1: Pricing a 100-Day Forward Contract on Dividend-Paying Stock



To calculate the **value of the long position in a forward contract on a dividend-paying stock**, we make the adjustment for the present value of the remaining expected discrete dividends at time *t* (PVD_t) to get:

$$V_t\,(\text{long position}) = [S_t - PVD_t] - \left[\frac{FP}{(1+R_f)^{(T-t)}}\right]$$

Professor's Note: This formula still looks like the standard "spot price minus present value of forward price." However, now the "spot price" has been adjusted by subtracting out the present value of the dividends because the long position in the forward contract does not receive the dividends paid on the underlying stock. So, now think "adjusted spot price less present value of forward price."

Example: Calculating the value of an equity forward contract on a stock

After 60 days, the value of the stock in the previous example is $36.00. Calculate the value of the equity forward contract on the stock to the long position, assuming the risk-free rate is still 5% and the yield curve is flat.

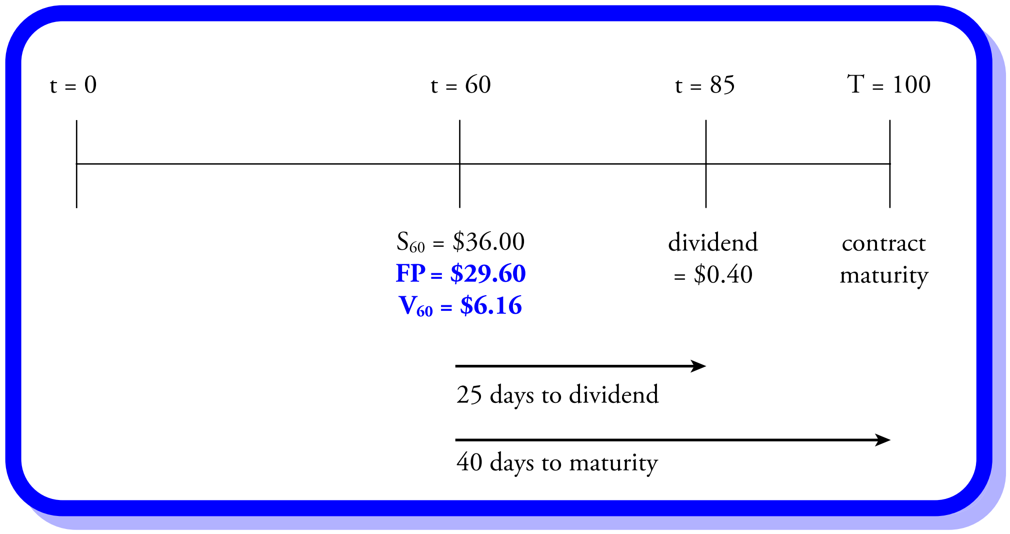
Answer:

There's only one dividend remaining (in 25 days) before the contract matures (in 40 days) as shown in Figure 2, so:

$$PVD_{60} = \frac{\$0.40}{1.05^{25/365}} = \$0.3987$$

$$V_{60}\,(\text{long position}) = \$36.00 - \$0.3987 - \left[\frac{\$29.60}{1.05^{40/365}}\right] = \$6.16$$

Figure 2: Valuing a 100-Day Forward Contract After 60 Days



To calculate the **price of an equity index forward contract**, rather than take the present value of each dividend on (possibly) hundreds of stocks, we can make the calculation as if the dividends are paid continuously (rather than at discrete times) at the dividend yield rate on the index. Using continuous time discounting, we can calculate the no-arbitrage forward price as:

$$\text{FP(on an equity index)} = S_0 \times e^{(R_f^c - \delta^c) \times T} = \left(S_0 \times e^{-\delta^c \times T}\right) \times e^{R_f^c \times T}$$

where:
R_f^c = continuously compounded risk-free rate
δ^c = continuously compounded dividend yield

Professor's Note: The relationship between the discrete risk-free rate R_f and the continuously compounded rate R_f^c is $R_f^c = \ln(1 + R_f)$. For example, 5% compounded annually is equal to ln(1.05) = 0.04879 = 4.879% compounded continuously. The 2-year 5% future value factor can then be calculated as either $1.05^2 = 1.1025$ or $e^{0.04879 \times 2} = 1.1025$.

Example: Calculating the price of a forward contract on an equity index

The value of the S&P 500 index is 1,140. The continuously compounded risk-free rate is 4.6% and the continuous dividend yield is 2.1%. Calculate the no-arbitrage price of a 140-day forward contract on the index.

Answer:

$$FP = 1{,}140 \times e^{(0.046 - 0.021) \times (140/365)} = 1{,}151$$

For the continuous time case, the **value of the forward contract on an equity index** is calculated as follows:

$$V_t\text{(of the long position)} = \left(\frac{S_t}{e^{\delta^c \times (T-t)}}\right) - \left(\frac{FP}{e^{R_f^c \times (T-t)}}\right)$$

Example: Calculating the value of a forward contract on an equity index

After 95 days, the value of the index in the previous example is 1,025. Calculate the value to the long position of the forward contract on the index, assuming the continuously compounded risk-free rate is 4.6% and the continuous dividend yield is 2.1%.

Answer:

After 95 days there are 45 days remaining on the original forward contract:

$$V_{95}\left(\text{of the long position}\right)=\left(\frac{1{,}025}{e^{0.021\times(45/365)}}\right)-\left(\frac{1{,}151}{e^{0.046\times(45/365)}}\right)=-122.14$$

Forward Contracts on Fixed Income Securities and Rates

LOS 64.d: Calculate and interpret the price and value of 1) a forward contract on a fixed income security, 2) a forward rate agreement (FRA), and 3) a forward contract on a currency.

In order to calculate the no-arbitrage **forward price on a coupon-paying bond**, we can use the same formula as we used for a dividend-paying stock or portfolio, simply substituting the present value of the expected coupon payments (PVC) *over the life of the contract* for the present value of the expected dividends to get the following formula:

$$FP\left(\text{on a fixed income security}\right)=\left(S_0-PVC\right)\times\left(1+R_f\right)^T$$

The value of the forward contract prior to expiration is as follows:

$$V_t\left(\text{long position}\right)=\left[S_t-PVC_t\right]-\left[\frac{FP}{\left(1+R_f\right)^{(T-t)}}\right]$$

In our examples, we assume that the spot price on the underlying coupon-paying bond includes accrued interest.

Example: Calculating the price of a forward on a fixed income security

Calculate the price of a 250-day forward contract on a 7% U.S. Treasury bond with a spot price of $1,050 (including accrued interest) that has just paid a coupon and will make another coupon payment in 182 days. The annual risk-free rate is 6%.

Answer:

Remember that U.S. Treasury bonds make semiannual coupon payments, so:

$$C=\frac{\$1{,}000\times 0.07}{2}=\$35.00$$

$$PVC=\frac{\$35.00}{1.06^{182/365}}=\$34.00$$

The forward price of the contract is therefore:

$$FP(\text{on a fixed income security}) = (\$1{,}050 - \$34.00) \times 1.06^{250/365} = \$1{,}057.37$$

Example: Calculating the value of a forward on a fixed income security

After 100 days, the value of the bond in the previous example is $1,090. Calculate the value of the forward contract on the bond to the long position, assuming the risk-free rate is 6.0%.

Answer:

There is only one coupon remaining (in 82 days) before the contract matures (in 150 days), so:

$$PVC = \frac{\$35.00}{1.06^{82/365}} = \$34.54$$

$$V_{100}(\text{long position}) = \$1{,}090 - \$34.54 - \left(\frac{\$1{,}057.37}{1.06^{150/365}}\right) = \$23.11$$

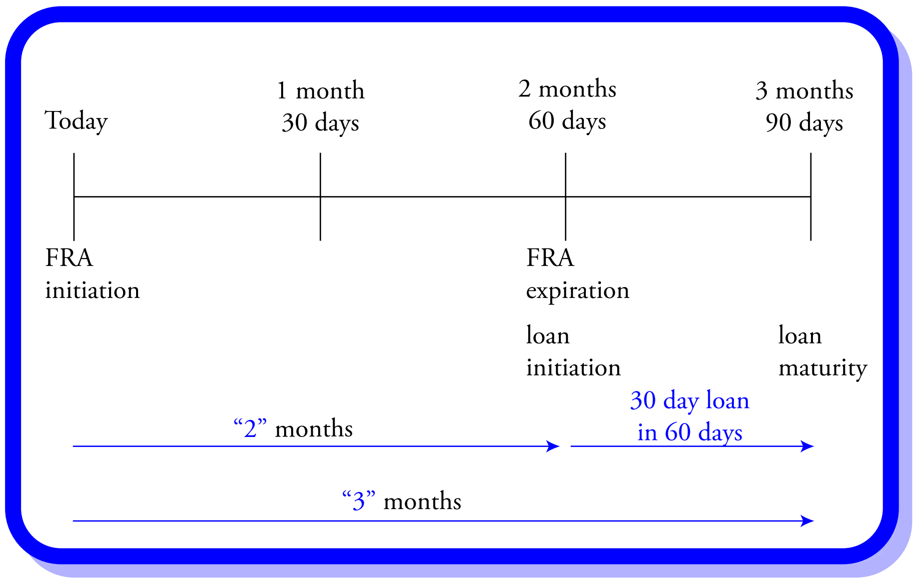
Warm-Up: Forward Rate Agreements

The long position in a **forward rate agreement** (FRA) is the party that would borrow the money (long the loan with the contract price being the interest rate on the loan). If the floating rate at contract expiration (LIBOR for U.S. dollar deposits and Euribor for Euro deposits) is above the rate specified in the forward agreement, the long position in the contract can be viewed as the right to borrow at below market rates and the long will receive a payment. If rates at the expiration date are below the then-current market rates, the short will receive a cash payment from the long. (The right to lend at *above* market rates would have a positive value.)

Professor's Note: We say "can be viewed as" because an FRA is settled in cash, so there is no requirement to lend or borrow the amount stated in the contract. For this reason the creditworthiness of the long position is not a factor in the interest rate on the FRA. However, to understand the pricing and calculation of value for an FRA, viewing the contract as a commitment to lend or borrow at a certain interest rate at a future date is helpful.

The notation for FRAs is unique. There are two numbers associated with an FRA: the number of months until the contract expires, and the number of months until the underlying loan is settled. The difference between these two is the maturity of the underlying loan. For example, a 2 × 3 FRA is a contract that expires in two months (60 days), and the underlying loan is settled in three months (90 days). The underlying rate is 1-month (30-day) LIBOR on a 30-day loan in 60 days. See Figure 3.

Figure 3: Illustration of a 2 × 3 FRA



The forward "price" in an FRA is actually a forward interest rate. The calculation of a forward interest rate is presented in Level 1 as the computation of forward rates from spot rates. We will illustrate this calculation with an example.

There are three important things to remember about FRAs when we're pricing and valuing them:

- LIBOR rates in the Eurodollar market are quoted on a 30/360 day basis.
- The long position in an FRA, in effect, is long the rate, so he wins when the rate increases.
- Although the interest on the underlying loan won't be paid until the end of the loan (e.g., in three months in Figure 3), the payoff on the FRA occurs at the expiration of the FRA (e.g., in two months). Therefore, the payoff on the FRA is the present value of the interest savings on the loan (e.g., discounted one month in Figure 3).

Example: Calculating the price of an FRA

Calculate the price of a 1 × 4 FRA (i.e., a 90-day loan, 30 days from now). The current 30-day rate is 4% and the 120-day rate is 5% (both of these are annualized money market rates).

Answer:

The actual (unannualized) rate on the 30-day loan is:

$$R30 = 0.04 \times \frac{30}{360} = 0.00333$$

The actual (unannualized) rate on the 120-day loan is:

$$R120 = 0.05 \times \frac{120}{360} = 0.01667$$

We wish to calculate the actual rate on a 90-day loan from day 30 to day 120:

$$\text{price of } 1 \times 4 \text{ FRA} = \frac{1 + R120}{1 + R30} - 1 = \frac{1.01667}{1.00333} - 1 = 0.0133$$

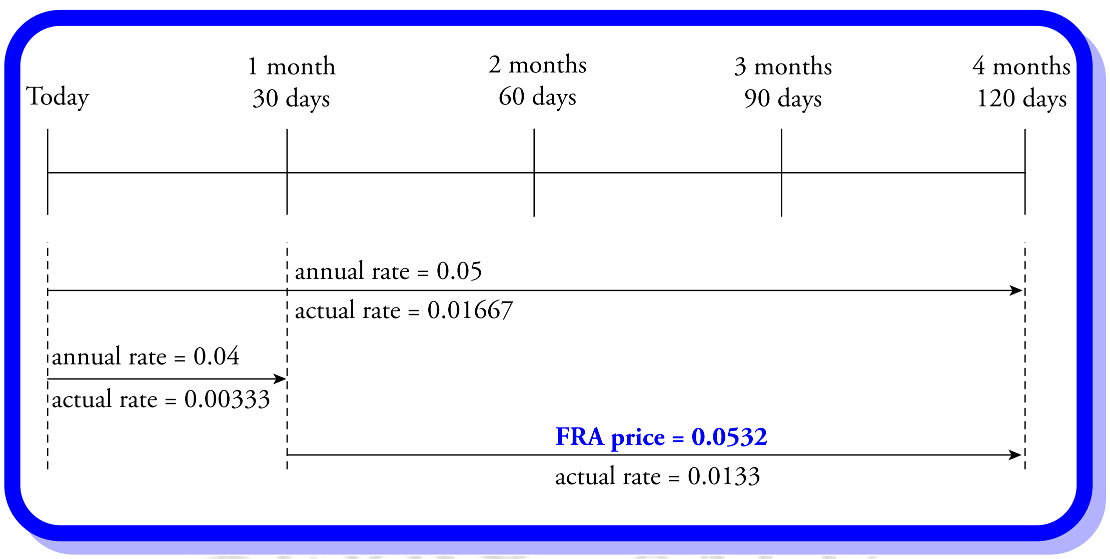
We can annualize this rate as:

$$0.0133 \times \frac{360}{90} = 0.0532 = 5.32\%$$

This is the no-arbitrage forward rate, the forward rate that will make the values of the long and the short positions in the FRA both zero at the initiation of the contract.

The time line is shown in Figure 4.

Figure 4: Pricing a 1 × 4 FRA



To understand the calculation of the value of the FRA *after the initiation of the contract*, recall that in the previous example the long in the FRA has the "right" to borrow money 30 days from inception for a period of 90 days at the forward rate. If interest rates increase (specifically the 90-day forward contract rate), the long will profit as the contract has fixed a borrowing rate below the now-current market rate. These "savings" will come at the end of the loan term, so to value the FRA we need to take the present value of these savings. An example incorporating this fact will illustrate the cash settlement value of an FRA at expiration.

Example: Calculating value of an FRA at maturity (i.e., cash payment at settlement)

Continuing the prior example for a 1 × 4 FRA, assume a notional principal of $1 million and that, at contract expiration, the 90-day rate has increased to 6%, which is above the contract rate of 5.32%. Calculate the value of the FRA at maturity, which is equal to the cash payment at settlement.

Answer:

The interest savings at the end of the loan term (compared to the market rate of 6%) will be:

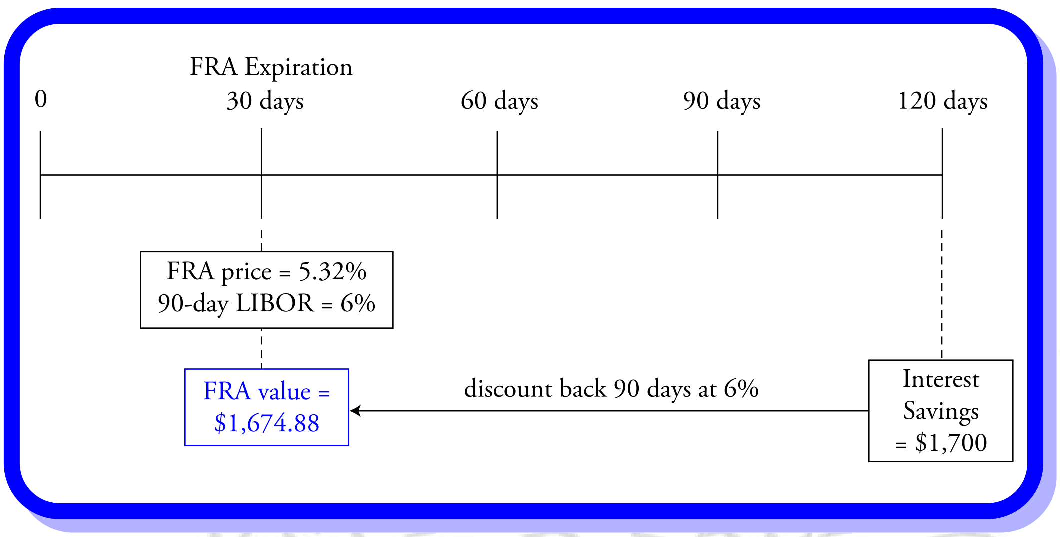
$$\left[\left(0.0600 \times \frac{90}{360}\right) - \left(0.0532 \times \frac{90}{360}\right)\right] \times \$1{,}000{,}000 = \$1{,}700$$

The present value of this amount at the FRA settlement date (90 days prior to the end of the loan term) discounted at the current rate of 6% is:

$$\frac{\$1{,}700}{1+\left(0.06\times\frac{90}{360}\right)}=\$1{,}674.88$$

This will be the cash settlement payment from the short to the long at the expiration of the contract. Note that we have discounted the savings in interest at the end of the loan term by the *market* rate of 6% that prevails at the contract settlement date for a 90-day term, as shown in Figure 5.

Figure 5: Valuing a 1 × 4 FRA at Maturity



To value an FRA prior to the settlement date, we need to know the number of days that have passed since the initiation of the contract. For example, let's suppose we want to value the same 1 × 4 FRA ten days after initiation. Originally it was a 1 × 4 FRA, which means the FRA now expires in 20 days. The calculation of the "savings" on the loan will be the same as in our previous example, except that we need to use the now-current market rate for a 90-day loan made at the settlement date, 20 days in the future. Also, we need to discount the interest savings implicit in the FRA back an extra 20 days, or 110 days, instead of 90 days as we did for the value at the settlement date. The time line is shown in Figure 6.

Example: Calculating value of an FRA prior to settlement

Value a 5.32% 1 × 4 FRA with a principal amount of $1 million 10 days after initiation if 110-day LIBOR is 5.9% and 20-day LIBOR is 5.7%.

Answer:

Step 1: Find the current 90-day forward rate at the settlement date, 20 days from now.

$$\left[\frac{1+\left(0.059\times\frac{110}{360}\right)}{1+\left(0.057\times\frac{20}{360}\right)}-1\right]\times\frac{360}{90}=0.0592568$$

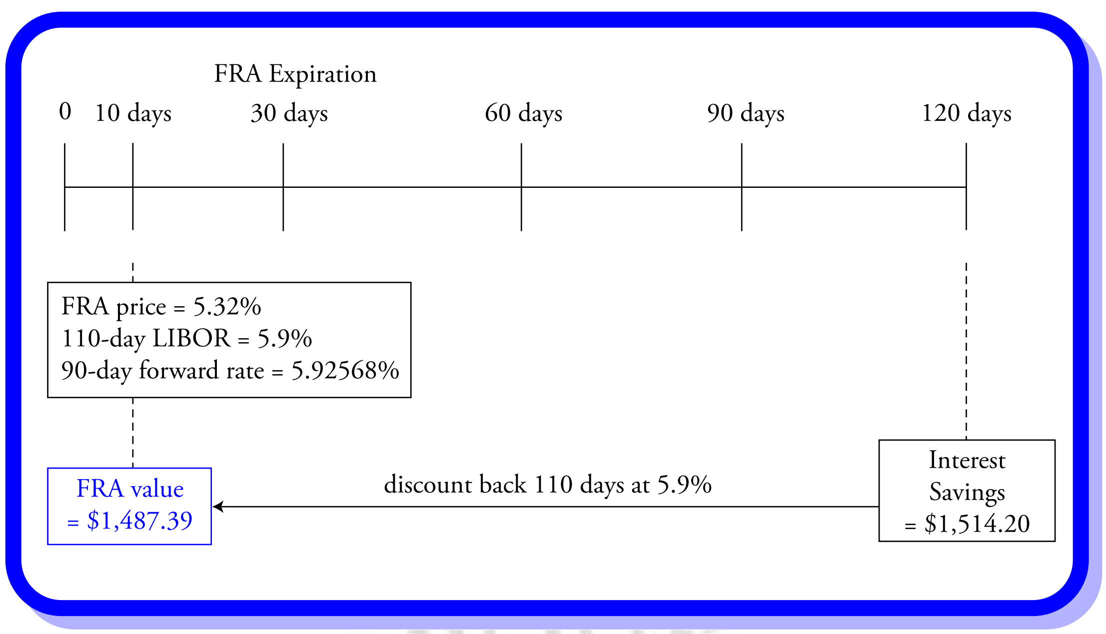
Step 2: Calculate the interest difference on a $1 million, 90-day loan made 20 days from now at the forward rate calculated above compared to the FRA rate of 5.32%.

$$\left[\left(0.0592568\times\frac{90}{360}\right)-\left(0.0532\times\frac{90}{360}\right)\right]\times \$1{,}000{,}000=\$1{,}514.20$$

Step 3: Discount this amount at the current 110-day rate.

$$\frac{\$1{,}514.20}{1+\left(0.059\times\frac{110}{360}\right)}=\$1{,}487.39$$

Figure 6: Valuing a 1 × 4 FRA Prior to Settlement



Professor's Note: I have tried to explain these calculations in such a way that you can value an FRA at any date from initiation to settlement using basic tools that you already know. Once you understand where the value of an FRA comes from (the interest savings on a loan to be made at the settlement date) and when this value is to be received (at the end of the loan), you can calculate the present value of these savings even under somewhat stressful test conditions. Just remember that if the rate in the future is less than the FRA rate, the long is "obligated to borrow" at above-market rates and will have to make a payment to the short. If the rate is greater than the FRA rate, the long will receive a payment from the short.

The **price and value of a currency forward contract** is refreshingly straightforward after that last bit of mental exercise. The calculation of the currency forward rate is just an application of covered interest parity from the topic review of foreign exchange parity relations in Study Session 4.

Recall that the interest rate parity result is based on an assumption that you should make the same amount when you lend at the riskless rate in your home country as you would if you bought *one unit* of the foreign currency at the current spot rate, S_0, invested it at the foreign risk-free rate, and entered into a forward contract to exchange the proceeds of the investment at maturity for the home currency at the forward rate of F_T (both the forward and the spot rates are quoted as the price in the home currency for one unit of the foreign currency).

Covered interest rate parity gives us the no-arbitrage forward price of a unit of foreign currency in terms of the home currency for a currency forward contract of length *T* in years:

$$F_T\,(\text{currency forward contract}) = S_0 \times \frac{(1+R_{DC})^T}{(1+R_{FC})^T}$$

where :
F and S are quoted in domestic currency per unit of foreign currency
R_{DC} = domestic currency interest rate
R_{FC} = foreign currency interest rate

Professor's Note: This is different from the way we expressed interest rate parity back in Study Session 4, in which F and S were quoted in terms of foreign currency per unit of domestic currency. The key is to remember our numerator/denominator rule: if the spot and forward quotes are in currency A per unit of currency B, the currency A interest rate should be on top and the currency B interest rate should be on the bottom. For example, if S and F are in euros per Swiss franc, put the European interest rate on the top and the Swiss interest rate on the bottom.

Example: Calculating the price of a currency forward contract

The risk-free rates are 6% in the U.S. and 8% in Mexico. The current spot exchange rate is \$0.0845 per Mexican peso (MXN). Calculate the forward exchange rate for a 180-day forward contract.

Answer:

$$F_T\,(\text{currency forward contract}) = \$0.0845 \times \frac{1.06^{180/365}}{1.08^{180/365}} = \$0.0837$$

At any time prior to maturity, the value of a currency forward contract to the long will depend on the spot rate at time *t*, S_t, and can be calculated as:

$$V_t\,(\text{currency forward contract}) = \left[\frac{S_t}{(1+R_{FC})^{(T-t)}}\right] - \left[\frac{F_T}{(1+R_{DC})^{(T-t)}}\right]$$

Example: Calculating the value of a currency forward contract

Calculate the value of the forward contract in the previous example if, after 15 days, the spot rate is \$0.0980 per MXN.

Answer:

$$V_{15}\,(\text{currency forward contract}) = \left(\frac{\$0.0980}{1.08^{165/365}}\right) - \left(\frac{\$0.0837}{1.06^{165/365}}\right) = \$0.0131$$

The continuous time price and value formulas for currency forward contracts are:

$$F_T = (\text{currency forward contract}) = S_0 \times e^{\left(R_{DC}^c - R_{FC}^c\right) \times T}$$

$$V_t\left(\text{currency forward contract}\right) = \left[\frac{S_t}{e^{R_{FC}^c \times (T-t)}}\right] - \left[\frac{F_T}{e^{R_{DC}^c \times (T-t)}}\right]$$

V_t in both cases is the value in domestic currency units for a contract covering one unit of the foreign currency. For the settlement payment in the home currency on a contract, simply multiply this amount by the notional amount of the foreign currency covered in the contract.

Credit Risk in Forward Contracts

LOS 64.e: Evaluate credit risk in a forward contract and how market value is a measure of the credit risk to a party in a forward contract.

At any date after initiation of a forward contract, it is likely to have positive value to either the long or the short. Recall that this value is the amount that would be paid to settle the contract in cash at that point in time. The party with the position that has positive value has credit risk in this amount because the other party would owe them that amount if the contract were terminated. The contract value and, therefore, the credit risk, may increase, decrease, or even change sign over the remaining term of the contract. However, at any point in time, the market values of forward contracts, as we have calculated them, are a measure of the credit risk currently borne by the party to which a cash payment would be made to settle the contract at that point. One way to reduce the credit risk in a forward contract is to mark-to-market part way through.

Key Concepts

1. The theoretical forward contract price for any asset is the price which would prevent a riskless arbitrage transaction under the assumption of no transactions costs, no credit risk, and unlimited borrowing and lending at the risk-free rate:

   $$FP = S_0 \times (1+R_f)^T \quad \text{or} \quad S_0 = \frac{FP}{(1+R_f)^T}$$

2. The value of the long position in a forward contract on an asset that makes no periodic payment during the term of the contract is the spot price of the asset minus the present value of the forward price. This can be positive or negative:

   $$V_0\left(\text{of long position at initiation}\right) = S_0 - \left[\frac{FP}{(1+R_f)^T}\right]$$

3. An off-market forward has a forward price at contract initiation that makes the value non-zero. Either the long or the short will receive a payment from the other party as compensation for entering into the contract.
4. The calculation of the forward price for an equity forward contract is different because the periodic dividend payments affect the no-arbitrage price calculation. The forward price is reduced by the future value of the expected dividend payments; alternatively the spot price is reduced by the present value of the dividends.

   $$FP\left(\text{on an equity security}\right) = (S_0 - PVD) \times (1+R_f)^T = \left[S_0 \times (1+R_f)^T\right] - FVD$$