All Kaplan Schweser topics are cross-referenced to the GARP assigned readings in the event that further clarification is needed.

The **sample variance**, s^2, is the measure of dispersion that applies when we are evaluating a sample of *n* observations from a population. The sample variance is calculated using the following formula:

$$s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

The most noteworthy difference from the formula for population variance is that the denominator for s^2 is n – 1, one less than the sample size n, where σ^2 uses the entire population size *N*. Another difference is the use of the sample mean, $\bar{X}$, instead of the population mean, μ. Based on the mathematical theory behind statistical procedures, the use of the entire number of sample observations, *n*, instead of n – 1 as the divisor in the computation of s^2, will systematically *underestimate* the population parameter, σ^2, particularly for small sample sizes. This systematic underestimation causes the sample variance to be what is referred to as a **biased estimator** of the population variance. Using n – 1 instead of *n* in the denominator, however, improves the statistical properties of s^2 as an estimator of σ^2. Thus, s^2, as expressed in the equation above, is considered to be an unbiased estimator of σ^2.

Schweser Study Notes provide valuable examples which assist with application of a given topic.

Example: Sample variance

Assume the data used in the preceding examples represents only a sample of the managers at a large investment firm. What is the sample variance of these returns?

Answer:

$$\bar{X} = \frac{[30+12+25+20+23]}{5} = 22\%$$

$$s^2 = \frac{\left[(30-22)^2 + (12-22)^2 + (25-22)^2 + (20-22)^2 + (23-22)^2\right]}{5-1} = 44.5\left(\%^2\right)$$

Thus, the sample variance of $44.5(\%^2)$ can be interpreted to be an unbiased estimator of the population variance.

As with the population standard deviation, the **sample standard deviation** can be calculated by taking the square root of the sample variance. The sample standard deviation, *s*, is defined as:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

Example: Sample standard deviation

Compute the sample standard deviation based on the result of the preceding example.

Answer:

Since the sample variance for the preceding example was computed to be 44.5(%^{2}), the sample standard deviation is:

$s = [44.5(\%^2)]^{1/2} = 6.67\%$

The results shown here mean that the sample standard deviation, s = 6.67%, can be interpreted as an unbiased estimator of the population standard deviation, σ.

Sample Covariance and Correlation

Clearly written formulas describe each variable within a given equation.

Sample covariance is calculated as:

$$\text{covariance} = \sum_{i=1}^{n} \frac{\left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{n-1}$$

where:
n = sample size
X_i = ith observation on the variable X
$\overline{X}$ = mean of the variable X observations
Y_i = ith obserbation on the variable Y
$\overline{Y}$ = mean of the variable Y observations

The actual value of the covariance is not very meaningful because its measurement is extremely sensitive to the scale of the two variables. Also, the covariance may range from negative to positive infinity and its computation results in squared units (e.g., percent squared). For these reasons, we calculate the correlation coefficient, which converts the covariance into something that is more useful and intuitively appealing.

Example: Sample covariance calculation

Calculate the covariance between the samples of stock returns for companies X and Y described in Figure 7.

Figure 1: Stock Returns for Companies X and Y

	Year					
Company	1	2	3	4	5	6
X	6%	8%	9%	9%	10%	9%
Y	5%	7%	9%	8%	9%	10%

Answer:

The easiest way to solve this problem is using the spreadsheet approach shown in Figure 8.

Figure 2: Calculating Sample Covariance of Stock Returns

Year	X_i	Y_i	$(X_i - \bar{X})$	$(Y_i - \bar{Y})$	$(X_i - \bar{X})(Y_i - \bar{Y})$
1	6	5	–2.5	–3.0	7.5
2	8	7	–0.5	–1.0	0.5
3	9	9	0.5	1.0	0.5
4	9	8	0.5	0.0	0.0
5	10	9	1.5	1.0	1.5
6	9	10	0.5	2.0	1.0
	$\bar{X} = 8.5$	$\bar{Y} = 8.0$			$\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y}) = 11$

$$\text{Cov}(X,Y) = \sum_{i=1}^{n}\frac{(X_i - \bar{X})(Y_i - \bar{Y})}{n-1} = \frac{11}{6-1} = 2.2$$

We have highlighted the terms and "buzzwords" that you should be familiar with on exam day.

The **sample correlation coefficient**, *r*, is a measure of the strength of the linear relationship (correlation) between two variables. The correlation coefficient is unitless; it is a "pure" measure of the tendency of two variables to move together.

The sample correlation coefficient for two variables, *X* and *Y*, is calculated as:

$$r_{XY} = \frac{\text{sample covariance of X and Y}}{(\text{sample standard deviation of X})(\text{sample standard deviation of Y})} = \frac{\text{Cov}(X,Y)}{(s_X)(s_Y)}$$

Skewness and Kurtosis

Schweser Study Notes include all GARP assigned learning objectives (i.e., AIMs) which are listed directly before a given concept is presented.

AIM 4.13: Define, calculate and interpret skewness and kurtosis.

A distribution is **symmetrical** if it is shaped identically on both sides of its mean. Distributional symmetry implies that intervals of losses and gains will exhibit the same frequency. For example, a symmetrical distribution with a mean return of zero will have losses in the –6% to –4% interval as frequently as it will have gains in the +4% to +6%

interval. The extent to which a returns distribution is symmetrical is important because the degree of symmetry tells analysts if deviations from the mean are more likely to be positive or negative.

Skewness, or skew, refers to the extent to which a distribution is not symmetrical. Nonsymmetrical distributions may be either positively or negatively skewed and result from the occurrence of outliers in the data set. **Outliers** are observations with extraordinarily large values, either positive or negative.

- A *positively skewed* distribution is characterized by many outliers in the upper region, or right tail. A positively skewed distribution is said to be skewed right because of its relatively long upper (right) tail.
- A *negatively skewed* distribution has a disproportionately large amount of outliers that fall within its lower (left) tail. A negatively skewed distribution is said to be skewed left because of its long lower tail.

Mean, Median, and Mode for a Nonsymmetrical Distribution

Skewness affects the **location of the mean, median, and mode** of a distribution as summarized in the following bulleted list.

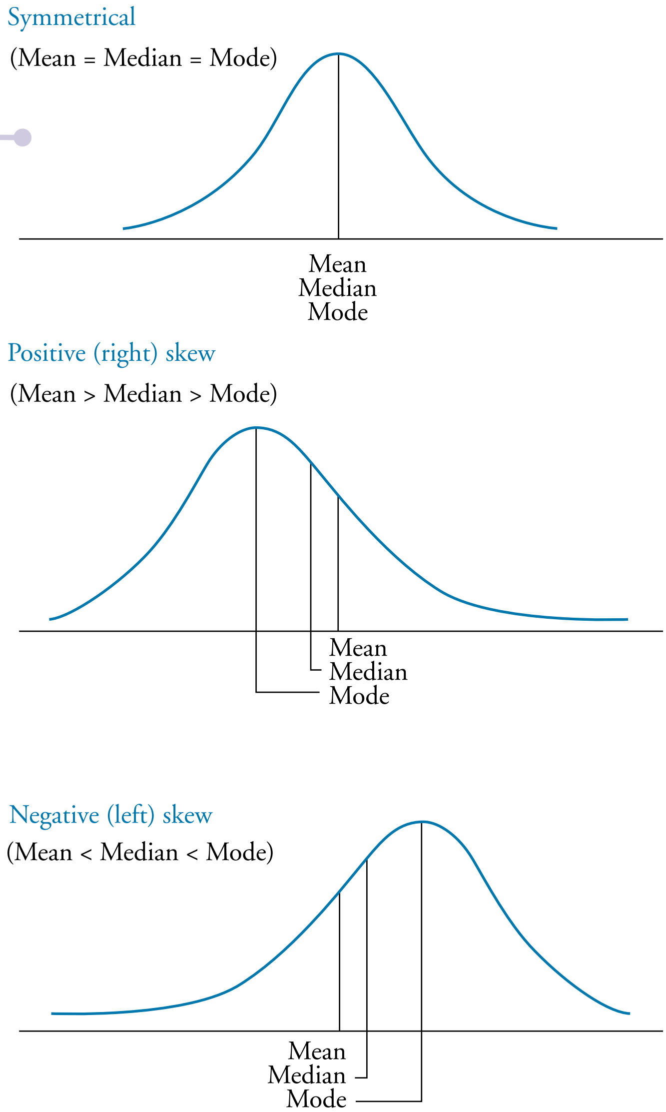
- For a symmetrical distribution, the mean, median, and mode are equal.
- For a positively skewed distribution, the mode is less than the median, which is less than the mean. The mean is affected by outliers; in a positively skewed distribution, there are large, positive outliers which will tend to "pull" the mean upward, or more positive. An example of a positively skewed distribution is that of housing prices. Suppose that you live in a neighborhood with 100 homes; 99 of them sell for $100,000, and one sells for $1,000,000. The median and the mode will be $100,000, but the mean will be $109,000. Hence, the mean has been "pulled" upward (to the right) by the existence of one home (outlier) in the neighborhood.
- For a negatively skewed distribution, the mean is less than the median, which is less than the mode. In this case, there are large, negative outliers which tend to "pull" the mean downward (to the left).

Professor's Note: The key to remembering how measures of central tendency are affected by skewed data is to recognize that skew affects the mean more than the median and mode, and the mean is "pulled" in the direction of the skew. The relative location of the mean, median, and mode for different distribution shapes is shown in Figure 9. Note the median is between the other two measures for positively or negatively skewed distributions.

Professor's Notes provide further analysis on a given concept and valuable insight on how topics are integrated throughout the curriculum.

You will find figures and tables, which help illustrate concepts for further understanding and retention.

Figure 3: Effect of Skewness on Mean, Median, and Mode



Kurtosis is a measure of the degree to which a distribution is more or less “peaked” than a normal distribution. **Leptokurtic** describes a distribution that is more peaked than a normal distribution, whereas **platykurtic** refers to a distribution that is less peaked, or flatter than a normal distribution.

As indicated in Figure 10, a leptokurtic return distribution will have more returns clustered around the mean and more returns with large deviations from the mean (fatter tails). Relative to a normal distribution, a leptokurtic distribution will have a greater percentage of small deviations from the mean and a greater percentage of extremely large deviations from the mean. This means that there is a relatively greater probability of an observed value being either close to the mean or far from the mean. With regard to an investment returns distribution, a greater likelihood of a large deviation from the mean return is often perceived as an increase in risk.

Key Concepts

Key concepts are provided for each assigned reading. These concepts summarize the material within a given topic.

1. Point estimates are single value estimates of population parameters, and confidence intervals are ranges of estimated values within which the actual value of the parameter will lie with a given probability.

2. Desirable statistical properties of an estimator include unbiasedness, efficiency, and consistency.

3. The *t*-distribution is similar, but not identical, to the normal distribution in shape—it is defined by the degrees of freedom, has a lower peak, and has fatter tails.

4. The *t*-distribution is used to construct confidence intervals for the population mean when the population variance is not known. The (1–α) confidence interval for the population mean, μ, is: $\overline{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$.

5. Use the *t*-distribution if:
   - Population distribution is normal with an unknown variance (large or small sample).
   - Population distribution is nonnormal with unknown variance, but the sample is large (n >30).

6. The standard normal distribution (*z*-distribution) is used to construct confidence intervals for the population mean when the population variance is known. The (1 – α) confidence interval for the population mean, μ, is: $\overline{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$.

7. Use the *z*-distribution if:
   - Population distribution is normal with known variance.
   - Population distribution is nonnormal and the sample is large ($n \geq 30$).

8. The hypothesis testing process requires a statement of a null and an alternative hypothesis, the selection of the appropriate test statistic, specification of the significance level, a decision rule, the calculation of a sample statistic, a decision regarding the hypotheses based on the test, and a decision based on the test results.

9. The null hypothesis is what the researcher wants to reject. The alternative hypothesis is what the researcher wants to prove, and it is accepted when the null hypothesis is rejected.

10. A two-tailed test results from a two-sided alternative hypothesis (e.g., H_A: $\mu \neq \mu_0$). A one-tailed test results from a one-sided alternative hypothesis (e.g., H_A: $\mu > \mu_0$, or H_A: $\mu < \mu_0$).

11. The decision rule depends on the alternative hypothesis and the distribution of the test statistic.

12. A Type I error is the rejection of the null hypothesis when it is actually true, while a Type II error is the failure to reject the null hypothesis when it is actually false.

13. The significance level can be interpreted as the probability that a test statistic will reject the null hypothesis by chance when it is actually true (i.e., the probability of a Type I error.)

14. The power of a test is the probability of rejecting the null when it is false. The power of a test = 1 – P(Type II error).

Concept Checkers

Concept checkers are provided for each assigned reading. These questions test a candidates knowledge of the material just reviewed and serve to assist with application of the main concepts. Answers to each question are provided at the end of each topic.

1. If the variance of the sampling distribution of an estimator is smaller than all other unbiased estimators of the parameter of interest, the estimator is:
   A. reliable.
   B. efficient.
   C. unbiased.
   D. consistent.

Use the following data to answer Questions 2 through 4.

Austin Roberts believes that the mean price of houses in the area is greater than \$145,000. A random sample of 36 houses in the area has a mean price of \$149,750. The population standard deviation is \$24,000, and Roberts wants to conduct a hypothesis test at a 1% level of significance.

2. The appropriate alternative hypothesis is:
   A. H_A: μ < \$145,000.
   B. H_A: μ ± \$145,000.
   C. H_A: $\mu \geq$ \$145,000.
   D. H_A: μ > \$145,000.

3. The value of the calculated test statistic is *closest* to:
   A. $z = 0.67$.
   B. $z = 1.19$.
   C. $z = 4.00$.
   D. $z = 8.13$.

4. Which of the following *most accurately* describes the appropriate test structure?
   A. *F*-test.
   B. Two-tailed test.
   C. One-tailed test.
   D. Chi-square test.

5. For a hypothesis test with a probability of a Type II error of 60% and a probability of a Type I error of 5%, which of the following statements is *most accurate*?
   A. The power of the test is 40%, and there is a 5% probability that the test statistic will exceed the critical value(s).
   B. There is a 95% probability that the test statistic will be between the critical values if this is a two-tail test.
   C. The power of the test is 55%, and the confidence level is 95%.
   D. There is a 5% probability that the null hypothesis will be rejected when actually true, and the probability of rejecting the null when it is false is 40%.

Supplement material is provided throughout our Schweser Study Notes. These supplemental readings provide background material on certain topics and assistance for those candidates who are unfamiliar with upcoming topics or need a refresher course in a given topic area.

Mechanics of Options Markets

The Valuation and Risk Analysis of Derivatives

Exam Focus

This review of options markets is an optional reading, which serves as background material for the upcoming topics related to options (Topics 16-22). The following material will benefit those candidates who are not familiar with the basic mechanics of options trading. Concepts discussed include strike prices, expiration dates, dividend and stock split adjustments, and position limits.

Underlying Assets

Exchange-traded options trade on four primary assets: (1) individual stocks, (2) foreign currency, (3) stock indices, and (4) futures. The market value of the underlying asset for foreign currency options will depend on the currency and the exchange rate. The market value of the underlying asset for futures options is the value of the underlying futures contract.

Stock Options Specifications

Expiration

Options can be either American or European style. **American options** can be exercised throughout the life of the option. **European options** can only be exercised on the expiration date of the option. For this reason, American options are always at least as valuable as corresponding European options. Exchange-traded stock options are American-style options. The expiration dates of these options dictate how the option is named. For example, a June put option on Intel means that the option expires in June. The actual day of expiration is the Saturday following the third Friday of the expiration month. Different expiration cycles dictate the actual expiration months of a stock option over a given year.

Strike Prices

Strike prices are dictated by the value of the stock. Low-value stocks have smaller strike increments than higher-value stocks. Typically, stocks that are priced around $20 have increments of $2.50, stocks that are priced around $50 have increments of $5.00, and so on. The strike price is usually denoted as "X" and the underlying stock as "S."

Moneyness, Time Value, and Intrinsic Value

An option class refers to all options of the same type, whether calls or puts. An option series refers to an option class with the same expiration. For a call (put), when the underlying asset price is less (greater) than the strike price, the option is said to be out of the money. For both a call and put, when the underlying asset price is equal to the strike price, the

Relevant past FRM exam questions are provided in the back of each book. These questions identify how concepts have been tested on previous FRM exams. GARP answers to each question are provided as well as Professor's Notes for additional clarification. The year a given question was tested and the topic tested from is also provided.

Old Exam Questions

Credit Risk Measurement and Management

1. Suppose the rate on 1-year zero-coupon corporate bonds is 13.5% and the implied probability of default is 3.96%. Assume LGD is 100%. Based on the given information, the 1-year T-bill rate is closest to:
   A. 4.49%.
   B. 9.00%.
   C. 6.74%.
   D. 6.00%.

2. Assume the marginal monthly default rates (conditional on no previous default) for a firm are 2% each month during the first year and 3% each month during the second year. What is the marginal probability of defaulting over the second year, conditional on not having defaulted the first year?
   A. Insufficient information to answer the question.
   B. 30.6%.
   C. 36.0%.
   D. 47.4%.

3. Assume that Akshaya Bank has a loan with a principal amount of USD 100 million outstanding to Brazil, due six months from now, and the loan has a present value of USD 100.51 million. Brazil declares its inability to meet its payment schedule and Akshaya Bank immediately negotiates a multi-year restructuring agreement with the following terms:

Principal Repayment:	Bullet to 2 years.
Loan Rate:	6% fixed, annual pay.
Upfront fee:	50 basis point.
Akshaya Bank's discount rate:	8%.
Guarantees and Options:	None.

   Based on the given information, Akshaya Bank's concessionality is close to:

   A. USD 96.93 million.
   B. USD 4.08 million.
   C. USD 96.43 million.
   D. USD 3.58 million.

4. The Thai default in 1997 was unusual compared to past sovereign defaults because:
   A. the country repudiated its debt, whereas most defaults are reschedulings.
   B. the country had a low inflation level, whereas most previous defaults had high inflation, largely as the result of fiscal deficits.
   C. the country had a strong banking system, whereas most previous defaults arose from weakness in the financial intermediation arena.
   D. the country was a strong exporter prior to the crisis, whereas most defaulting countries were net importers.

The index at the back of each book provides a list of key terms to be familiar with for the exam. Use the index for a quick reference if you are searching for a particular concept.

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