

The **sample variance**, s^2 , is the measure of dispersion that applies when we are evaluating a sample of n observations from a population. The sample variance is calculated using the following formula:

$$s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}$$

The most noteworthy difference from the formula for population variance is that the denominator for s^2 is $n - 1$, one less than the sample size n , where σ^2 uses the entire population size N . Another difference is the use of the sample mean, \bar{X} , instead of the population mean, μ . Based on the mathematical theory behind statistical procedures, the use of the entire number of sample observations, n , instead of $n - 1$ as the divisor in the computation of s^2 , will systematically *underestimate* the population parameter, σ^2 , particularly for small sample sizes. This systematic underestimation causes the sample variance to be what is referred to as a **biased estimator** of the population variance. Using $n - 1$ instead of n in the denominator, however, improves the statistical properties of s^2 as an estimator of σ^2 . Thus, s^2 , as expressed in the equation above, is considered to be an unbiased estimator of σ^2 .

All Kaplan Schweser topics are cross-referenced to the GARP assigned readings in the event that further clarification is needed.

Example: Sample variance

Assume the data used in the preceding examples represents only a sample of the managers at a large investment firm. What is the sample variance of these returns?

Answer:

$$\bar{X} = \frac{[30 + 12 + 25 + 20 + 23]}{5} = 22\%$$

$$s^2 = \frac{[(30 - 22)^2 + (12 - 22)^2 + (25 - 22)^2 + (20 - 22)^2 + (23 - 22)^2]}{5 - 1} = 44.5(\%^2)$$

Thus, the sample variance of $44.5(\%^2)$ can be interpreted to be an unbiased estimator of the population variance.

The Schweser Study Notes provide valuable examples which assist with the application of a given topic.

As with the population standard deviation, the **sample standard deviation** can be calculated by taking the square root of the sample variance. The sample standard deviation, s , is defined as:

$$s = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}}$$

Example: Sample standard deviation

Compute the sample standard deviation based on the result of the preceding example.

Answer:

Since the sample variance for the preceding example was computed to be 44.5(%²), the sample standard deviation is:

$$s = [44.5(\%^2)]^{1/2} = 6.67\%$$

The results shown here mean that the sample standard deviation, $s = 6.67\%$, can be interpreted as an unbiased estimator of the population standard deviation, σ .

Sample Covariance and Correlation

Sample covariance is calculated as:

$$\text{covariance} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}$$

Clearly written formulas describe each variable within a given equation.

where:

n = sample size

X_i = i th observation on the variable X

\bar{X} = mean of the variable X observations

Y_i = i th observation on the variable Y

\bar{Y} = mean of the variable Y observations

The actual value of the covariance is not very meaningful because its measurement is extremely sensitive to the scale of the two variables. Also, the covariance may range from negative to positive infinity and its computation results in squared units (e.g., percent squared). For these reasons, we calculate the correlation coefficient, which converts the covariance into something that is more useful and intuitively appealing.

Example: Sample covariance calculation

Calculate the covariance between the samples of stock returns for companies X and Y described in Figure 7.

Figure 7: Stock Returns for Companies X and Y

	Year					
Company	1	2	3	4	5	6
X	6%	8%	9%	9%	10%	9%
Y	5%	7%	9%	8%	9%	10%

Answer:

The easiest way to solve this problem is using the spreadsheet approach shown in Figure 8.

Figure 8: Calculating Sample Covariance of Stock Returns

Year	X_i	Y_i	$(X_i - \bar{X})$	$(Y_i - \bar{Y})$	$(X_i - \bar{X})(Y_i - \bar{Y})$
1	6	5	-2.5	-3.0	7.5
2	8	7	-0.5	-1.0	0.5
3	9	9	0.5	1.0	0.5
4	9	8	0.5	0.0	0.0
5	10	9	1.5	1.0	1.5
6	9	10	0.5	2.0	1.0

$\bar{X} = 8.5$	$\bar{Y} = 8.0$	$\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) = 11$
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$$\text{Cov}(X,Y) = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1} = \frac{11}{6 - 1} = 2.2$$

The **sample correlation coefficient**, r , is a measure of the strength of the linear relationship (correlation) between two variables. The correlation coefficient is unitless; it is a “pure” measure of the tendency of two variables to move together.

The sample correlation coefficient for two variables, X and Y , is calculated as:

$$r_{XY} = \frac{\text{sample covariance of X and Y}}{(\text{sample standard deviation of X})(\text{sample standard deviation of Y})} = \frac{\text{Cov}(X, Y)}{(s_X)(s_Y)}$$

We have highlighted the terms that you should be familiar with on exam day.

SKEWNESS AND KURTOSIS

AIM 11.7: Define, calculate and interpret the skewness and kurtosis of a random variable.

AIM 11.8: Describe and identify a platykurtic and leptokurtic distribution.

AIM 11.9: Define the skewness and kurtosis of a normally distributed random variable.

A distribution is **symmetrical** if it is shaped identically on both sides of its mean. Distributional symmetry implies that intervals of losses and gains will exhibit the same frequency. For example, a symmetrical distribution with a mean return of zero will have losses in the -6% to -4% interval as frequently as it will have gains in the $+4\%$ to $+6\%$ interval. The extent to which a returns distribution is symmetrical is important because the degree of symmetry tells analysts if deviations from the mean are more likely to be positive or negative.

Skewness, or skew, refers to the extent to which a distribution is not symmetrical. Nonsymmetrical distributions may be either positively or negatively skewed and result from the occurrence of outliers in the data set. **Outliers** are observations with extraordinarily large values, either positive or negative.

- A *positively skewed* distribution is characterized by many outliers in the upper region, or right tail. A positively skewed distribution is said to be skewed right because of its relatively long upper (right) tail.
- A *negatively skewed* distribution has a disproportionately large amount of outliers that fall within its lower (left) tail. A negatively skewed distribution is said to be skewed left because of its long lower tail.

Mean, Median, and Mode for a Nonsymmetrical Distribution

Skewness affects the **location of the mean, median, and mode** of a distribution as summarized in the following bulleted list.

- For a symmetrical distribution, the mean, median, and mode are equal.
- For a positively skewed distribution, the mode is less than the median, which is less than the mean. The mean is affected by outliers; in a positively skewed distribution, there are large, positive outliers which will tend to “pull” the mean upward, or more positive. An example of a positively skewed distribution is that of housing prices. Suppose that you live in a neighborhood with 100 homes; 99 of them sell for \$100,000, and one sells for \$1,000,000. The median and the mode will be \$100,000, but the mean will be \$109,000. Hence, the mean has been “pulled” upward (to the right) by the existence of one home (outlier) in the neighborhood.
- For a negatively skewed distribution, the mean is less than the median, which is less than the mode. In this case, there are large, negative outliers which tend to “pull” the mean downward (to the left).

The Schweser Study Notes include all GARP assigned learning objectives (i.e., AIMS) which are listed directly before a given concept is presented.



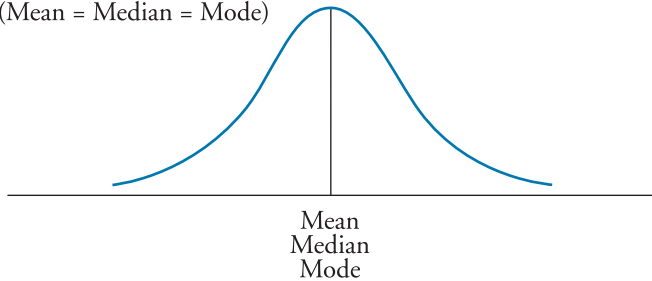
Professor's Note: The key to remembering how measures of central tendency are affected by skewed data is to recognize that skew affects the mean more than the median and mode, and the mean is "pulled" in the direction of the skew. The relative location of the mean, median, and mode for different distribution shapes is shown in Figure 9. Note the median is between the other two measures for positively or negatively skewed distributions.

Professor's Notes provide further analysis on a given concept and valuable insight on how topics are integrated throughout the curriculum.

Figure 9: Effect of Skewness on Mean, Median, and Mode

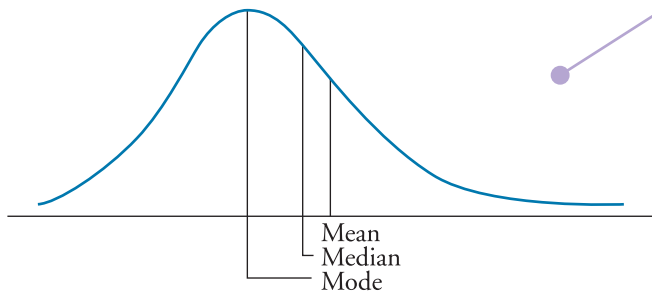
Symmetrical

(Mean = Median = Mode)



Positive (right) skew

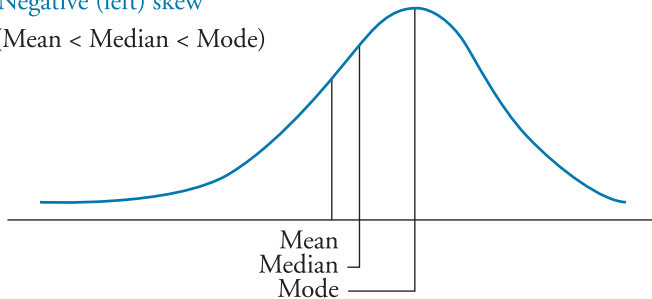
(Mean > Median > Mode)



You will find figures and tables, which help illustrate concepts for further understanding and retention.

Negative (left) skew

(Mean < Median < Mode)



Kurtosis is a measure of the degree to which a distribution is more or less “peaked” than a normal distribution. **Leptokurtic** describes a distribution that is more peaked than a normal distribution, whereas **platykurtic** refers to a distribution that is less peaked, or flatter than a normal distribution.

As indicated in Figure 10, a leptokurtic return distribution will have more returns clustered around the mean and more returns with large deviations from the mean (fatter tails). Relative to a normal distribution, a leptokurtic distribution will have a greater percentage

KEY CONCEPTS

- Point estimates are single value estimates of population parameters, and confidence intervals are ranges of estimated values within which the actual value of the parameter will lie with a given probability.
- Desirable statistical properties of an estimator include unbiasedness, efficiency, and consistency.
- The t -distribution is similar, but not identical, to the normal distribution in shape—it is defined by the degrees of freedom, has a lower peak, and has fatter tails.
- The t -distribution is used to construct confidence intervals for the population mean when the population variance is not known. The $(1-\alpha)$ confidence interval for the population mean, μ , is: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$.
- Use the t -distribution if:
 - Population distribution is normal with an unknown variance (large or small sample).
 - Population distribution is nonnormal with unknown variance, but the sample is large ($n > 30$).
- The standard normal distribution (z -distribution) is used to construct confidence intervals for the population mean when the population variance is known. The $(1-\alpha)$ confidence interval for the population mean, μ , is: $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$.
- Use the z -distribution if:
 - Population distribution is normal with known variance.
 - Population distribution is nonnormal and the sample is large ($n \geq 30$).
- The hypothesis testing process requires a statement of a null and an alternative hypothesis, the selection of the appropriate test statistic, specification of the significance level, a decision rule, the calculation of a sample statistic, a decision regarding the hypotheses based on the test, and a decision based on the test results.
- The null hypothesis is what the researcher wants to reject. The alternative hypothesis is what the researcher wants to prove, and it is accepted when the null hypothesis is rejected.
- A two-tailed test results from a two-sided alternative hypothesis (e.g., $H_A: \mu \neq \mu_0$). A one-tailed test results from a one-sided alternative hypothesis (e.g., $H_A: \mu > \mu_0$, or $H_A: \mu < \mu_0$).
- The decision rule depends on the alternative hypothesis and the distribution of the test statistic.
- A Type I error is the rejection of the null hypothesis when it is actually true, while a Type II error is the failure to reject the null hypothesis when it is actually false.
- The significance level can be interpreted as the probability that a test statistic will reject the null hypothesis by chance when it is actually true (i.e., the probability of a Type I error).
- The power of a test is the probability of rejecting the null when it is false. The power of a test = $1 - P$ (Type II error).

Key concepts are provided for each assigned reading. These concepts summarize the material within a given topic.

CONCEPT CHECKERS

- If the variance of the sampling distribution of an estimator is smaller than all other unbiased estimators of the parameter of interest, the estimator is:
 - reliable.
 - efficient.
 - unbiased.
 - consistent.

Concept checkers are provided for each assigned reading. These questions test a candidate's knowledge of the material just reviewed and serve to assist with application of the main concepts. Answers to each question are provided at the end of each topic.

Use the following data to answer Questions 2 through 4.

Austin Roberts believes that the mean price of houses in the area is greater than \$145,000. A random sample of 36 houses in the area has a mean price of \$149,750. The population standard deviation is \$24,000, and Roberts wants to conduct a hypothesis test at a 1% level of significance.

- The appropriate alternative hypothesis is:
 - $H_A: \mu < \$145,000$.
 - $H_A: \mu \pm \$145,000$.
 - $H_A: \mu \geq \$145,000$.
 - $H_A: \mu > \$145,000$.
- The value of the calculated test statistic is *closest* to:
 - $z = 0.67$.
 - $z = 1.19$.
 - $z = 4.00$.
 - $z = 8.13$.
- Which of the following *most accurately* describes the appropriate test structure?
 - F -test.
 - Two-tailed test.
 - One-tailed test.
 - Chi-square test.
- For a hypothesis test with a probability of a Type II error of 60% and a probability of a Type I error of 5%, which of the following statements is *most accurate*?
 - The power of the test is 40%, and there is a 5% probability that the test statistic will exceed the critical value(s).
 - There is a 95% probability that the test statistic will be between the critical values if this is a two-tail test.
 - The power of the test is 55%, and the confidence level is 95%.
 - There is a 5% probability that the null hypothesis will be rejected when actually true, and the probability of rejecting the null when it is false is 40%.

3. Based on the regression results, which model is more reliable?
- The passenger car model because $3.86 > 1.93$.
 - The motorcycle model because $1.93 < 3.86$.
 - The passenger car model because $0.758 > 0.676$.
 - The motorcycle model because $\sqrt{0.676} < \sqrt{0.758}$.
4. Which of the following is not a necessary assumption of simple linear regression analysis?
- The residuals are normally distributed.
 - There is a constant variance of the error term.
 - The independent variable is uncorrelated with the residuals.
 - The dependent variable is uncorrelated with the residuals.
5. What is the appropriate alternative hypothesis to test the statistical significance of the intercept term in the following regression?
- $$Y = a_1 + a_2(X) + \varepsilon$$
- $H_A: a_1 \neq 0$.
 - $H_A: a_1 > 0$.
 - $H_A: a_2 \neq 0$.
 - $H_A: a_2 > 0$.

CHALLENGE PROBLEMS

6. In testing a set of data, you are trying to determine if the normality assumption is reasonable using a 1% significance level. The sample is considered large at 150 observations, and you have determined that the skewness and the excess kurtosis of the data are 0.5 and 0.2, respectively.

Based on the information provided above, which of the following statements are correct?

- Jarque-Bera value is 6.5.
 - Jarque-Bera value is 55.25.
 - Reject the null hypothesis that the data is normal.
 - Not reject the null hypothesis that the data is normal.
- I and III.
 - I and IV.
 - II and III.
 - II and IV.

Challenge Problems are designed to provide candidates with additional questions that rigorously test their understanding of the core FRM concepts.

Relevant past FRM exam questions are provided in the back of each book. These questions identify how concepts have been tested on previous FRM exams. GARP answers to each question are provided as well as Professor's Notes for additional clarification. The year a given question was tested and the topic tested from is also provided.

FRM OLD QUESTION REVIEW (FROM PREVIOUS FRM EXAMS)

Credit Risk Measurement and Management



Professor's Note: The following questions are from the 2006, 2004, 2003, and 2002 actual FRM exams.

1. Suppose the rate on 1-year zero-coupon corporate bonds is 13.5% and the implied probability of default is 3.96%. Assume LGD is 100%. Based on the given information, the 1-year T-bill rate is closest to:
 - A. 4.49%.
 - B. 9.00%.
 - C. 6.74%.
 - D. 6.00%.

2. Which of the following is not a modeling approach to credit scoring?
 - A. k-nearest neighbor classifier models.
 - B. Logit and Probit models.
 - C. Fisher linear discriminant analysis.
 - D. Bayesian vector autoregression.

3. Which of the following is not a commonly used method for generating a recovery rate function?
 - A. Nonparametric kernel estimation.
 - B. Cubic SPLINE estimation.
 - C. Assume the recovery rate follows a beta distribution.
 - D. Estimate conditional densities with generalized method of moments.

4. Which of the following reduce a credit exposure by shortening the effective maturity of a position?
 - I. Liquidity put.
 - II. Credit trigger.
 - A. Both I and II.
 - B. I but not II.
 - C. II but not I.
 - D. Neither of I or II.

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