Study Session 16: Analysis of Fixed Income Investments—Analysis and Valuation

Introduction to the Valuation of Debt Securities

Cross-Reference to CFA Institute Assigned Reading #65

Bond Valuation Process

- Estimate the cash flows over the life of the security—coupon payments and return of principal.
- Determine the appropriate discount rate based on risk associated with the estimated cash flows.
- Calculate the present value of the estimated cash flows.

Difficulties in Estimating the Expected Cash Flows

- Timing of principal repayments is not known with certainty.
- Coupon payments are not known with certainty.
- The bond is convertible or exchangeable into another security.

Bond Valuation

Bond prices, established in the market, can be expressed either as a percentage of par value or as a yield. Yield to maturity (YTM) is the single discount rate that will make the present value of a bond's promised semiannual cash flows equal to the market price.

In the United States, bonds typically make coupon payments (equal to one-half the stated coupon rate times the face value) twice a year, and the yield to maturity is expressed as twice the semiannual discount rate that will make the present value of the semiannual coupon payments equal to the market price. This yield to maturity, calculated for a semiannual-pay bond, is also referred to as a *bond equivalent yield.*

For bonds that make annual payments, the YTM is the annual discount rate that makes the present value of the annual coupon payments equal to the market price. Thus, the relation between an annual and semiannual YTM is:

$$YTM_{\text{annual-pay}} = \left(1 + \frac{YTM_{\text{semiannual-pay}}}{2}\right)^2 - 1$$

$$YTM_{\text{semiannual-pay}} = \left[(1 + YTM_{\text{annual-pay}})^{\frac{1}{2}} - 1\right] \times 2$$

The relation between the semiannual YTM (the bond equivalent yield) and price for a bond with *N* years to maturity can be represented as:

$$\text{bond price} = \frac{CPN_1}{(1 + YTM/2)} + \frac{CPN_2}{(1 + YTM/2)^2} + \ldots + \frac{CPN_{2N} + Par}{(1 + YTM/2)^{2N}}$$

The price-yield relationship for a zero-coupon bond with *N* years to maturity is based on a semiannual yield or bond equivalent yield by convention, so we have:

$$\text{zero-coupon bond price} = \frac{\text{face value}}{\left(1 + \frac{YTM}{2}\right)^{2N}}$$

$$\text{zero-coupon YTM} = \left[\left(\frac{\text{face value}}{\text{price}}\right)^{\frac{1}{2N}} - 1\right] \times 2$$

A bondholder will actually realize the YTM on his initial investment only if all payments are made as scheduled, the bond is held to maturity, and, importantly, all interim cash flows are reinvested at the YTM.

Spot rates and no-arbitrage bond values:

Earlier we discussed a yield curve that plotted YTM versus bond maturity. We can call that the “par yield curve” if it is constructed with the YTMs for bonds trading at par.

Spot rates are market discount rates for single payments to be received in the future and can be thought of, theoretically, as equivalent to the market yields on zero-coupon bonds. Given the spot-rate yield curve, we can discount each of a bond's promised cash flows at its appropriate spot rate and sum the resulting present values to get the market value of the bond.

Values calculated in this way are called *no-arbitrage values* and we will present the reason for this terminology shortly. With C_N and S_N being the *N*-period coupon payments and spot rates respectively, we can write:

$$\text{bond value} = \frac{C_1}{(1+S_1)^1} + \frac{C_2}{(1+S_2)^2} + \ldots\ldots \frac{C_N + \text{face}}{(1+S_N)^N}$$

Government bond dealers can separate Treasury bonds into their "pieces," the individual coupon and principal cash flows, a process known as *stripping the bond*. These individual pieces are a series of zero-coupon bonds with different maturity dates, and each can be valued by discounting at the spot rate for the appropriate maturity. Since bond dealers can also recombine a bond's individual cash flows into a bond, arbitrage prevents the market price of the bond from being more or less than the value of the individual cash flows discounted at spot rates.

If the spot-rate or no-arbitrage value is greater than the market price, a bond dealer can buy the bond, strip it, and sell the "pieces" for the greater amount to earn an arbitrage profit. If the market price of the bond is greater than the no-arbitrage value, a dealer can buy the pieces, combine them into a bond, and sell the bond to make a profit.

Yield Measures, Spot Rates, and Forward Rates

Cross-Reference to CFA Institute Assigned Reading #66

This chapter includes information on many different types of yield measures. You must be ready to calculate any of them quickly and accurately.

Sources of Bond Return

- Periodic coupon interest payments.
- Recovery of principal, along with any capital gain or loss.
- Reinvestment income.

Traditional Measures of Yield

Current yield:

$$\text{current yield} = \frac{\text{annual coupon payment}}{\text{bond price}}$$

This measure looks at just one source of return: *a bond's annual interest income*—it does not consider capital gains/losses or reinvestment income.

The relationships between different yield measures are displayed in the following table:

Bond selling at	*Relationship*
Par	coupon rate = current yield = yield to maturity
Discount	coupon rate < current yield < yield to maturity
Premium	coupon rate > current yield > yield to maturity

Yield to maturity, call, put, refunding:

Yields to other events besides maturity are calculated in the same way as YTM and are essentially internal rate of return measures. For example, to calculate the yield to call (YTC), we need to use the number of semiannual periods until the call date under consideration (for N) and the call price in place of the maturity value (for FV).

The key to YTC computations is using the right number of periods (to first call) and the appropriate terminal value (the call price).

Bootstrapping Spot Rates

Understand the concept of bootstrapping spot rates from coupon bond prices using known short-term spot rates.

Example:

A 2-year bond with an 8% annual coupon is priced at 100 and the 1-year spot rate is 4%. Use the bootstrapping method to find the 2-year spot rate.

Answer:

The arbitrage-free pricing relationship is $100 = \frac{8}{1.04} + \frac{108}{(1+Z_2)^2}$, so

we can write $100 - 7.6923 = \frac{108}{(1+Z_2)^2}$ and solve for Z_2 as

$$Z_2 = \left[\frac{108}{92.3077}\right]^{\frac{1}{2}} - 1 = 8.167\%.$$

The idea of bootstrapping is that we can repeat this process sequentially. Given Z_1, Z_2, and the price of a 3-year bond, we could calculate Z_3 in the same manner.

Forward Rates

A forward rate is a rate for borrowing/lending at some date in the future. The key here is that investors should receive the same total return from investing in a 2-year bond as they would if they invested in a 1-year bond and then rolled the proceeds into a second 1-year bond at maturity of the first bond, one year from today.

This idea is shown in the relation between an N-period spot rate and a series of forward 1-year rates. Letting ${}_1f_0$ be the current 1-year rate and ${}_1f_N$ be the 1-year rate *N* years from now, we can write:

$$\text{N-period spot rate } (S_N) = \left[(1 + {}_1f_0)(1 + {}_1f_1).......(1 + {}_1f_N)\right]^{\frac{1}{N+1}} - 1$$

The formula for computing the 1-period forward rate *n* periods from today using spot rates is:

$${}_1f_n = \frac{(1+\text{spot}_{n+1})^{n+1}}{(1+\text{spot}_n)^n} - 1$$

The Option-Adjusted Spread (OAS) and Zero-Volatility (Z) Spreads

The *nominal spread* is simply an issue's YTM minus the YTM of a Treasury security of similar maturity. Therefore, the use of the nominal spread suffers from the same limitations as the YTM.

The *static spread* (or *zero-volatility spread*) is not the spread over a single Treasury's YTM, but the spread over each of the spot rates on the spot rate yield curve. In other words, *the same spread is added to all risk-free spot rates* to make the PV of the bond's promised cash flows equal to its market price. The *Z*-spread is inherently more accurate than (and will usually differ from) the nominal spread, since it is based upon the arbitrage-free spot rates, rather than a given YTM. The *Z*-spread will equal the nominal spread if the term structure of interest rates (the yield curve) is perfectly flat.

The *option-adjusted spread* is used when a bond has embedded options. It can be thought of as the difference between the static or *Z*-spread and the option cost.

Z-spread – option adjusted spread = option cost in %

For a bond with a call feature, the option cost will be positive (you require a higher yield). For a bond with a put feature, the option cost will be negative since a bond with a put feature will have a lower required yield than an identical option-free bond.

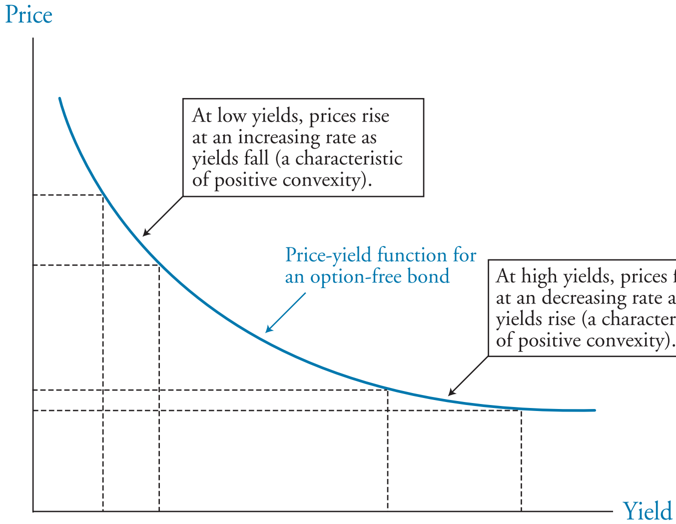
The intuition of the OAS is that it is the spread once any differences in yield due to the embedded option are removed. Thus, it is a spread that reflects differences in yield for differences in credit risk and liquidity. That's why it must be used for bonds with embedded options and will be the same as the *Z*-spread for option-free bonds.

Introduction to the Measurement of Interest Rate Risk

Cross-Reference to CFA Institute Assigned Reading #67

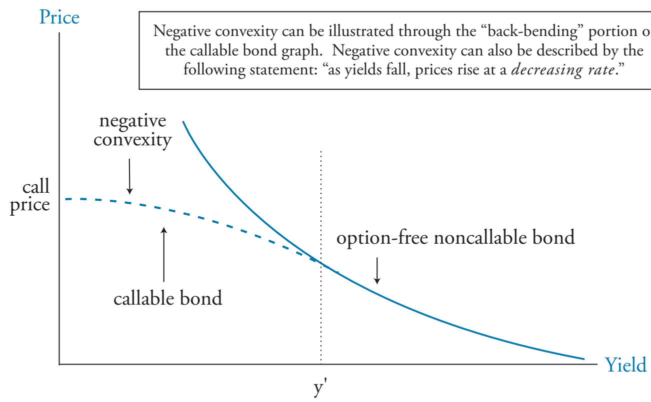
Duration is a measure of the *slope* of the price-yield function, which is steeper at low interest rates and flatter at high interest rates. Hence, duration (interest rate sensitivity) is higher at low rates and lower at high rates. This concept holds for non-callable bonds. Convexity is a measure of the degree of curvature of the price/yield relationship. Convexity accounts for the error in the estimated change in a bond's price based on duration.

Figure 2: Price-Yield Function of an Option-Free Bond



If the bond is callable and the bond is likely to be called, as yields fall, no one will pay a price higher than the call price. The price will not rise significantly as yields fall and you will see *negative convexity* at work. Remember, the verbal description of negative convexity is, "as yields fall, prices rise at a decreasing rate." For a positively convex bond, as yields fall, prices rise at an *increasing* rate.

Figure 3: Price-Yield Function of a Callable Bond



Measuring Interest Rate Risk

There are two approaches to measuring interest rate risk: the full valuation approach (scenario analysis approach) and the duration/convexity approach.

Full valuation or scenario analysis approach:

This approach revalues all bonds in a portfolio under a given interest rate change (yield curve) scenario. It is theoretically preferred and gives a good idea of the change in portfolio value. This method requires accurate valuation models and consists of these steps:

1. Start with current market yield and price.
2. Estimate changes in yields.
3. Revalue bonds.
4. Compare new value to current value.

Duration/convexity approach:

This approach provides an approximation of the actual interest rate sensitivity of a bond or bond portfolio. It has an advantage due to its simplicity compared to the full valuation approach.

The most concise, useful description of duration is that it represents *the sensitivity of a bond's (or portfolio's) price to a 1% change in yield to maturity.*

Know this formula for effective duration and be able to make computations with this formula, entering Δy as a decimal (e.g., 0.005 for one-half percent).

$$\text{effective duration} = \frac{\text{value when yield falls by } \Delta y - \text{value when yield rises by } \Delta y}{2 \times \text{beginning value} \times (\Delta y)}$$

The preceding equation provides a measure which allows us to approximate the percentage change in the price of a bond for a 100 basis point (1.00%) change in yield to maturity.

Modified duration assumes that the cash flows on the bond will not change (i.e., that we are dealing with a noncallable bond). This differs from *effective duration*, which considers expected changes in cash flows that may occur for bonds with embedded options. Effective duration must be used for bonds with embedded options.

Modified duration and *effective duration* are good approximations of potential bond price behavior only for relatively small changes in interest rates. As rate changes grow larger, the curvature of the bond price/yield relationship becomes more important. The widening error in the estimated price is due to the curvature of the actual price path, a bond's *convexity*.

Finally, it is critical that you know how to compute the approximate percentage price change of a bond. Use the decimal change in yield here, too.

percentage change in price = duration effect + convexity effect
$= [-\text{duration} \times (\Delta y)] + [\text{convexity} \times (\Delta y)^2] \times 100$

The *price value of a basis point* (PVBP) is the dollar change in a portfolio or asset value for one basis point change in yield.

PVBP = duration × 0.0001 × value