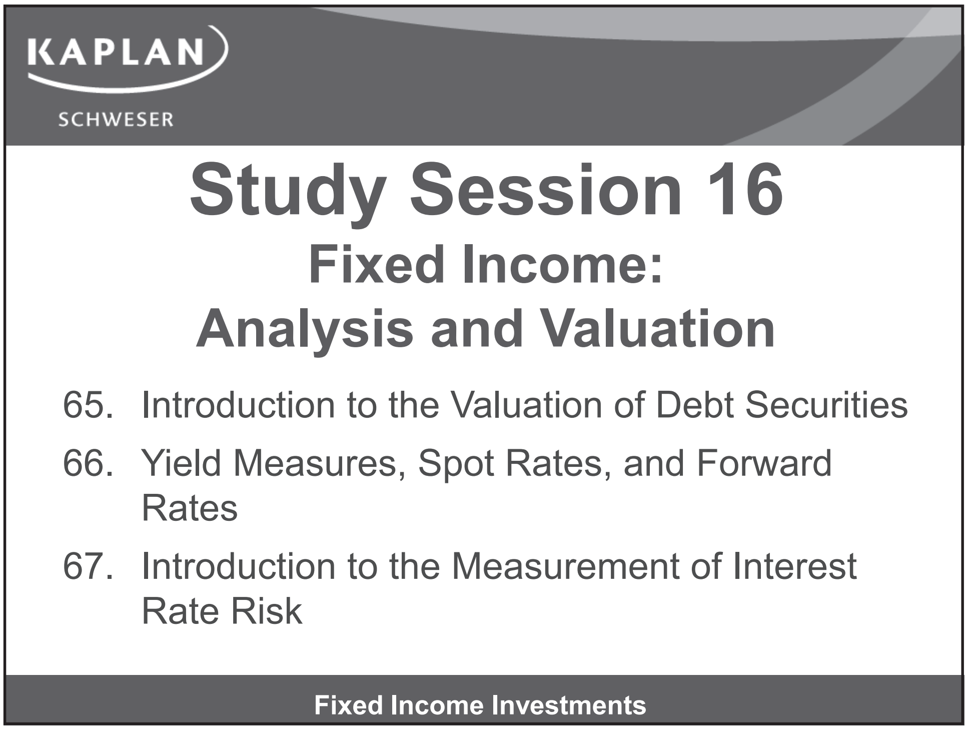
Study Session 16

Fixed Income: Analysis and Valuation



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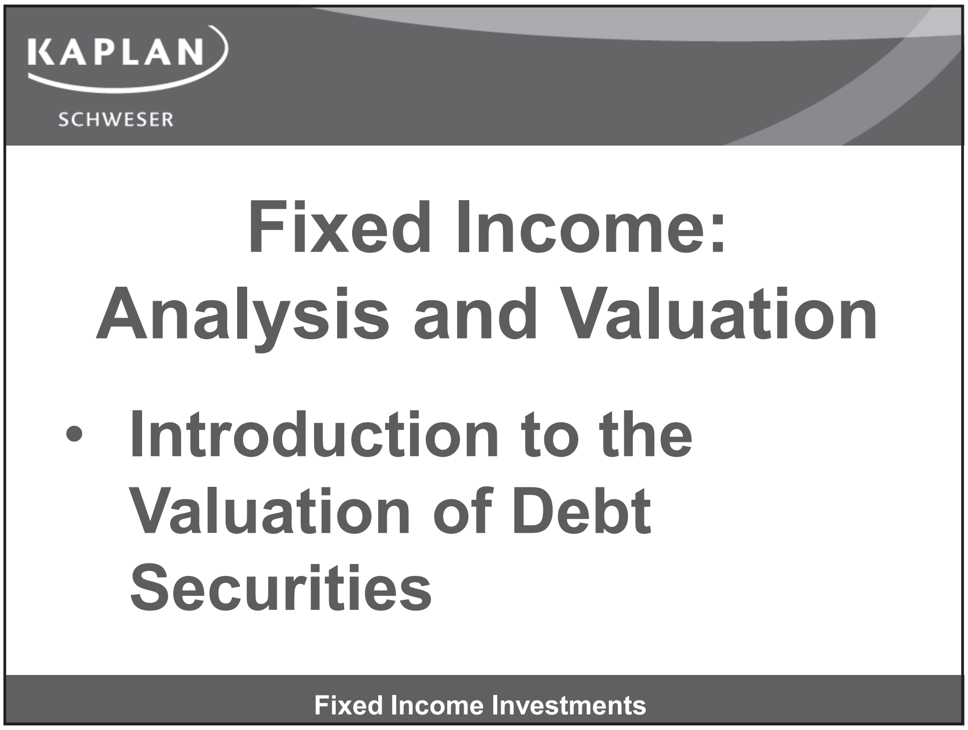
Study Session 16

Fixed Income:
Analysis and Valuation

65. Introduction to the Valuation of Debt Securities
66. Yield Measures, Spot Rates, and Forward Rates
67. Introduction to the Measurement of Interest Rate Risk

Fixed Income Investments



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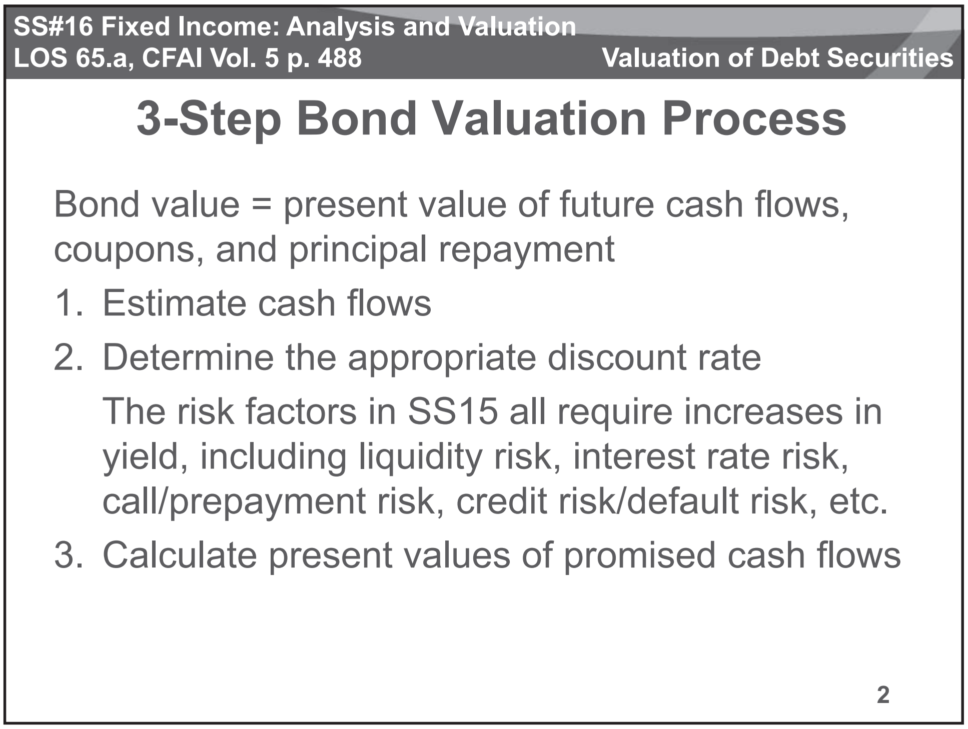
Fixed Income: Analysis and Valuation

- **Introduction to the Valuation of Debt Securities**

Fixed Income Investments



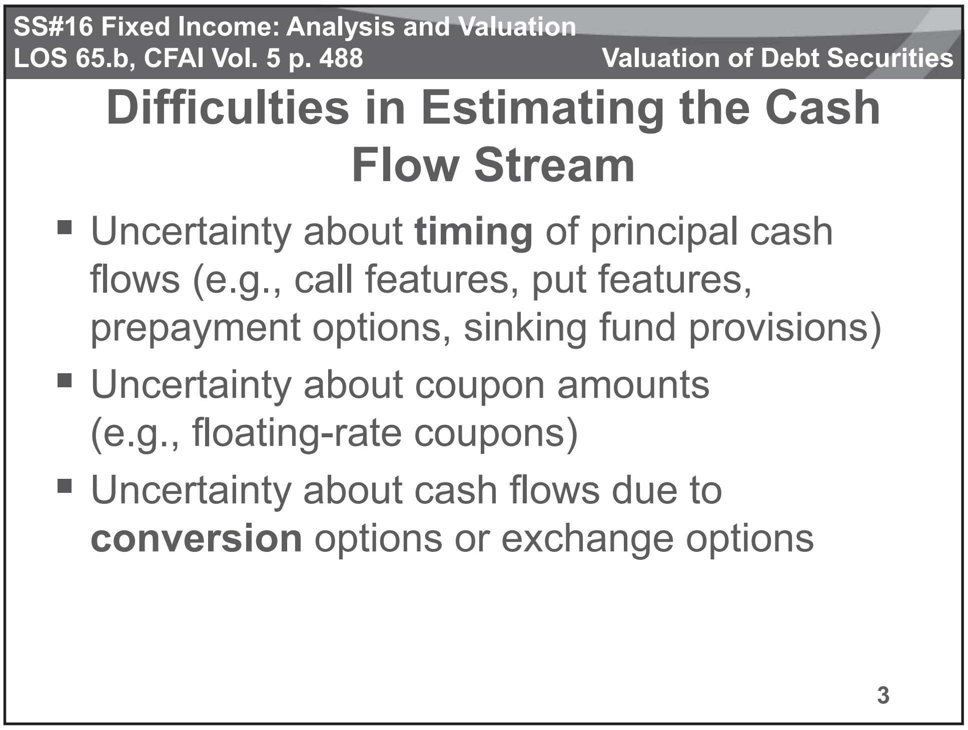
SS#16 Fixed Income: Analysis and Valuation
LOS 65.a, CFAI Vol. 5 p. 488 **Valuation of Debt Securities**

3-Step Bond Valuation Process

Bond value = present value of future cash flows, coupons, and principal repayment

1. Estimate cash flows
2. Determine the appropriate discount rate

   The risk factors in SS15 all require increases in yield, including liquidity risk, interest rate risk, call/prepayment risk, credit risk/default risk, etc.
3. Calculate present values of promised cash flows



SS#16 Fixed Income: Analysis and Valuation
LOS 65.b, CFAI Vol. 5 p. 488 **Valuation of Debt Securities**

Difficulties in Estimating the Cash Flow Stream

- Uncertainty about **timing** of principal cash flows (e.g., call features, put features, prepayment options, sinking fund provisions)
- Uncertainty about coupon amounts (e.g., floating-rate coupons)
- Uncertainty about cash flows due to **conversion** options or exchange options

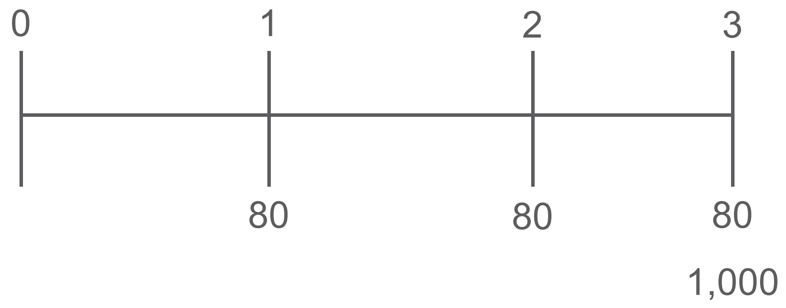
SS#16 Fixed Income: Analysis and Valuation
LOS 65.c, CFAI Vol. 5 p. 490 **Valuation of Debt Securities**

Valuing an Annual-Pay Bond Using a Single Discount Rate

- Term to maturity = 3 years
- Par = $1,000
- Coupon = 8% annual coupon
- Discount rate 12%

SS#16 Fixed Income: Analysis and Valuation
LOS 65.c, CFAI Vol. 5 p. 490 **Valuation of Debt Securities**

8% Annual-Pay Bond Cash Flows



SS#16 Fixed Income: Analysis and Valuation
LOS 65.c, CFAI Vol. 5 p. 490 **Valuation of Debt Securities**

Bond Value: 8% Coupon, 12% Yield

$$\frac{80}{(1.12)^1}+\frac{80}{(1.12)^2}+\frac{80+1{,}000}{(1.12)^3}=903.933$$

N = 3; I/Y = 12; PMT = 80; FV = 1,000;

CPT PV = $903.93

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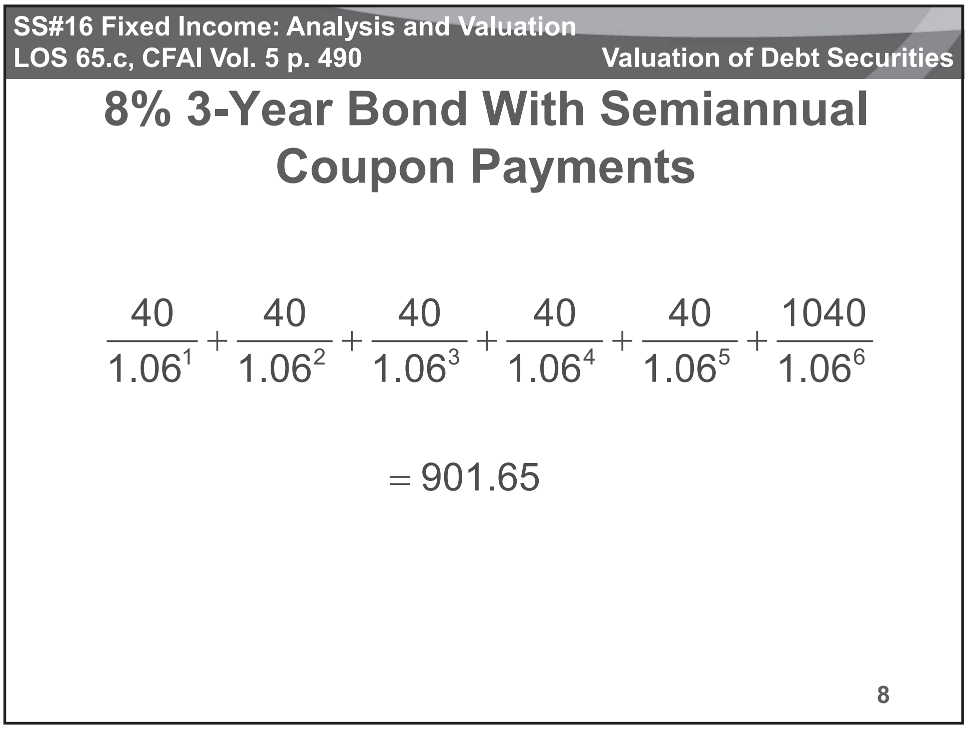
LOS 65.c, CFAI Vol. 5 p. 490 **Valuation of Debt Securities**

Same (8% 3-yr.) Bond With a Semiannual-Pay Coupon

PMT = coupon / 2 = $80 / 2 = $40
N = 2 × # of years to maturity = 3 × 2 = 6
I/Y = discount rate / 2 = 12 / 2 = 6%
FV = par = $1,000

N = 6; I/Y = 6; PMT = 40; FV = 1,000;
CPT PV = –901.65

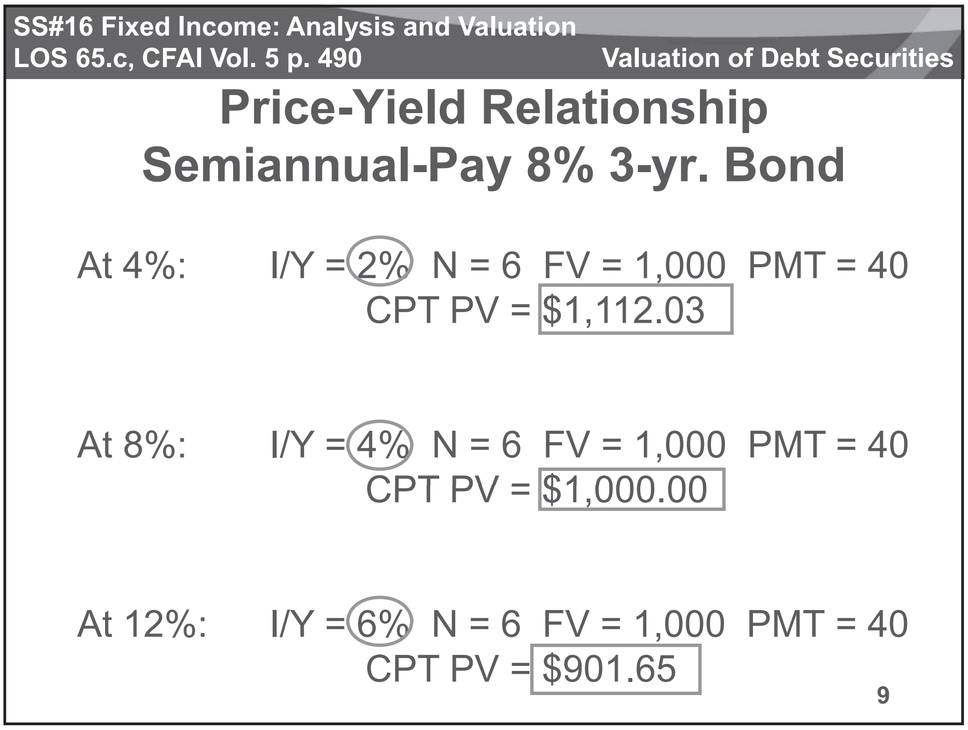


SS#16 Fixed Income: Analysis and Valuation
LOS 65.c, CFAI Vol. 5 p. 490 **Valuation of Debt Securities**

8% 3-Year Bond With Semiannual Coupon Payments

$$\frac{40}{1.06^1}+\frac{40}{1.06^2}+\frac{40}{1.06^3}+\frac{40}{1.06^4}+\frac{40}{1.06^5}+\frac{1040}{1.06^6}$$

$$=901.65$$



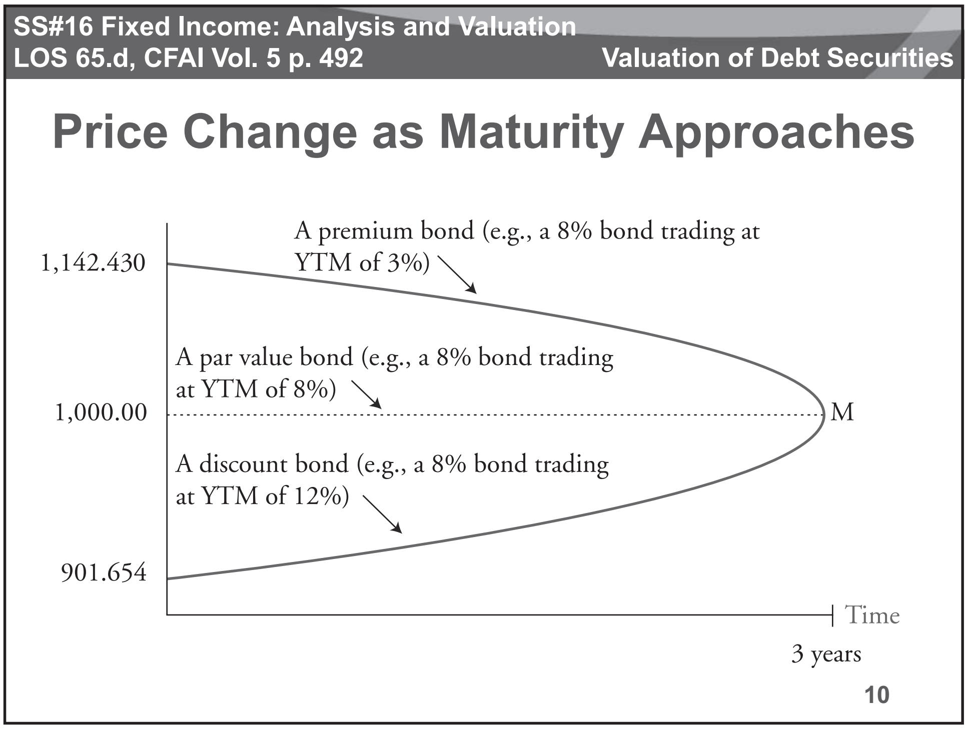
SS#16 Fixed Income: Analysis and Valuation
LOS 65.c, CFAI Vol. 5 p. 490 **Valuation of Debt Securities**

Price-Yield Relationship Semiannual-Pay 8% 3-yr. Bond

At 4%: I/Y = 2% N = 6 FV = 1,000 PMT = 40
CPT PV = $\boxed{\$1,112.03}$

At 8%: I/Y = 4% N = 6 FV = 1,000 PMT = 40
CPT PV = $\boxed{\$1,000.00}$

At 12%: I/Y = 6% N = 6 FV = 1,000 PMT = 40
CPT PV = $\boxed{\$901.65}$



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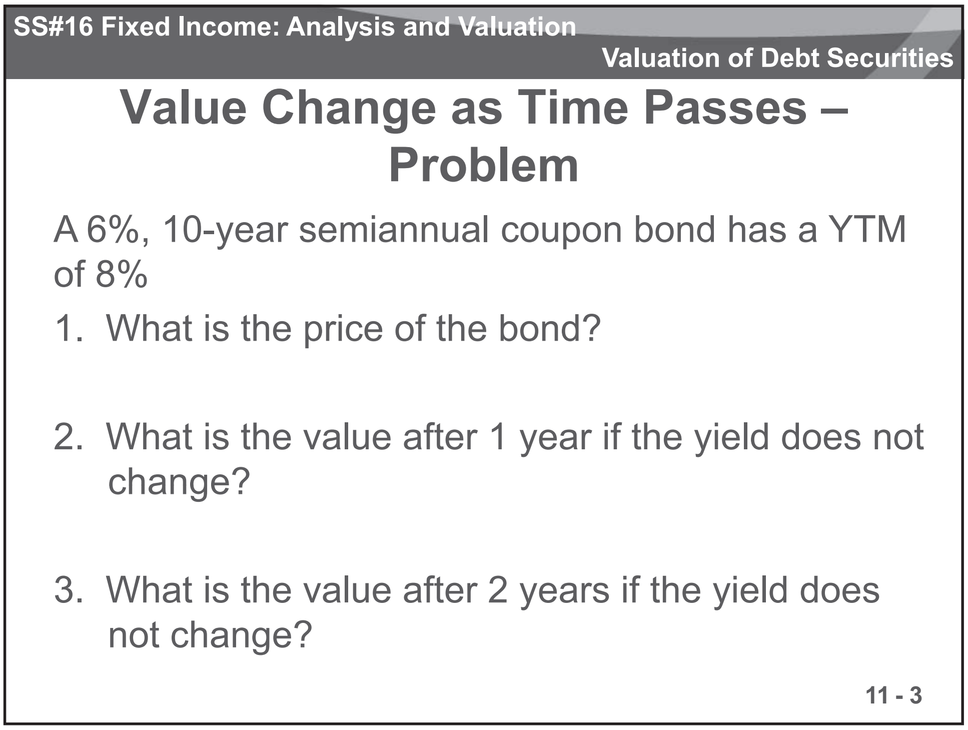
LOS 65.d, CFAI Vol. 5 p. 492
Valuation of Debt Securities

Price Change as Maturity Approaches



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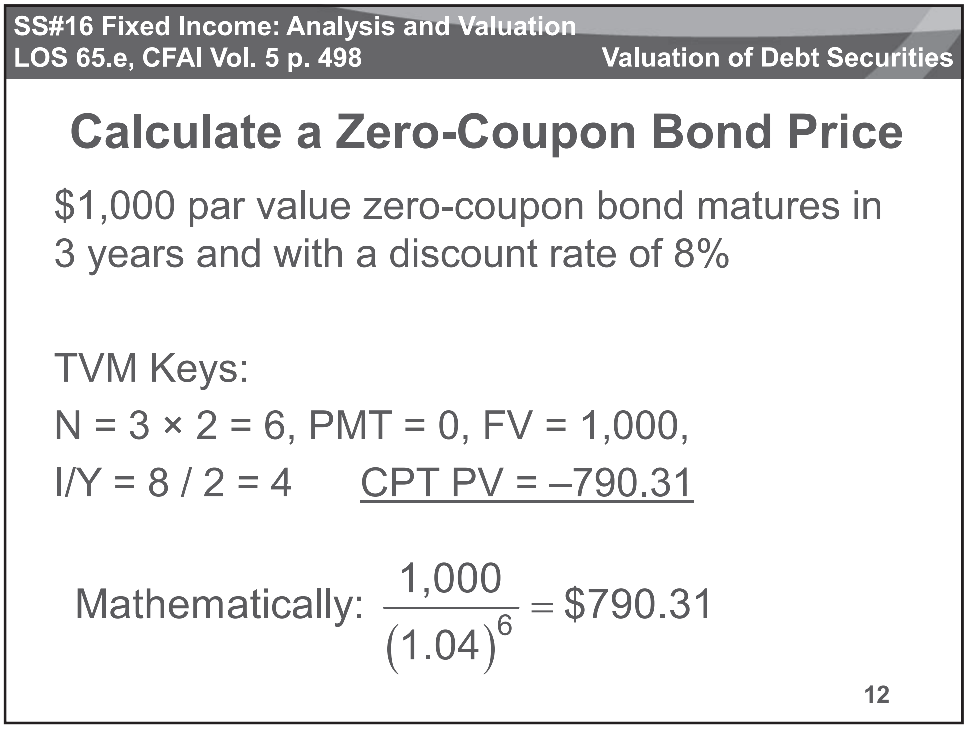
Valuation of Debt Securities

Value Change as Time Passes – Problem

A 6%, 10-year semiannual coupon bond has a YTM of 8%

1. What is the price of the bond?
2. What is the value after 1 year if the yield does not change?
3. What is the value after 2 years if the yield does not change?



SS#16 Fixed Income: Analysis and Valuation
LOS 65.e, CFAI Vol. 5 p. 498 **Valuation of Debt Securities**

Calculate a Zero-Coupon Bond Price

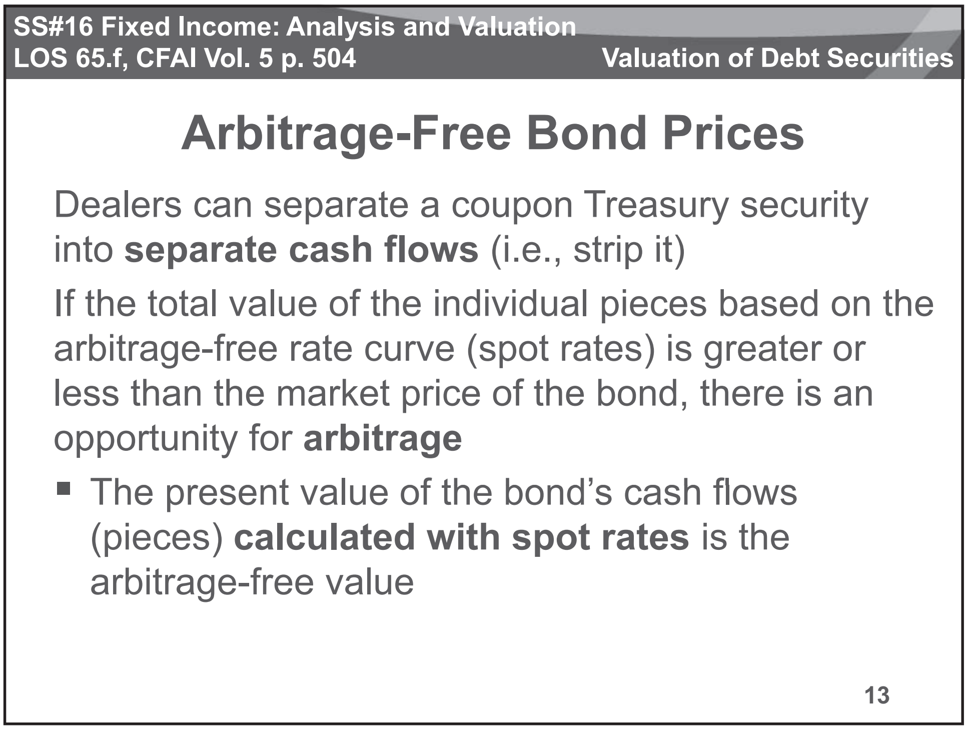
$1,000 par value zero-coupon bond matures in 3 years and with a discount rate of 8%

TVM Keys:

N = 3 × 2 = 6, PMT = 0, FV = 1,000,

I/Y = 8 / 2 = 4 <u>CPT PV = –790.31</u>

Mathematically: $$\frac{1{,}000}{(1.04)^{6}} = \$790.31$$



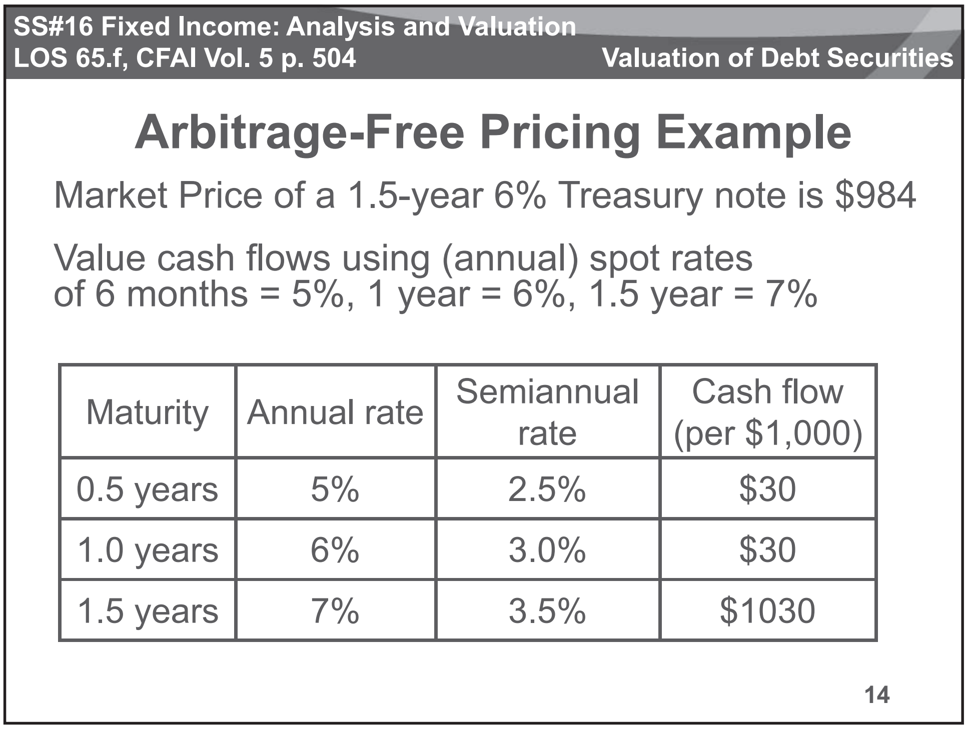
SS#16 Fixed Income: Analysis and Valuation
LOS 65.f, CFAI Vol. 5 p. 504 **Valuation of Debt Securities**

Arbitrage-Free Bond Prices

Dealers can separate a coupon Treasury security into **separate cash flows** (i.e., strip it)

If the total value of the individual pieces based on the arbitrage-free rate curve (spot rates) is greater or less than the market price of the bond, there is an opportunity for **arbitrage**

- The present value of the bond’s cash flows (pieces) **calculated with spot rates** is the arbitrage-free value



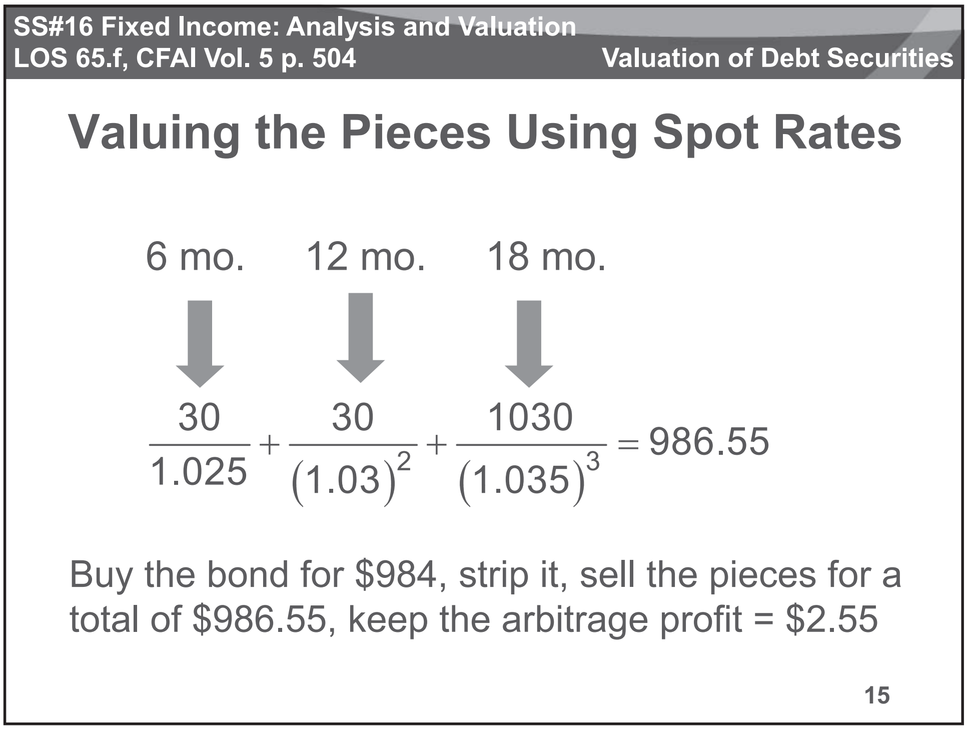
SS#16 Fixed Income: Analysis and Valuation
LOS 65.f, CFAI Vol. 5 p. 504 **Valuation of Debt Securities**

Arbitrage-Free Pricing Example

Market Price of a 1.5-year 6% Treasury note is $984

Value cash flows using (annual) spot rates of 6 months = 5%, 1 year = 6%, 1.5 year = 7%

Maturity	Annual rate	Semiannual rate	Cash flow (per $1,000)
0.5 years	5%	2.5%	$30
1.0 years	6%	3.0%	$30
1.5 years	7%	3.5%	$1030



SS#16 Fixed Income: Analysis and Valuation
LOS 65.f, CFAI Vol. 5 p. 504 **Valuation of Debt Securities**

Valuing the Pieces Using Spot Rates

$$\frac{30}{1.025}+\frac{30}{(1.03)^2}+\frac{1030}{(1.035)^3}=986.55$$

Buy the bond for $984, strip it, sell the pieces for a total of $986.55, keep the arbitrage profit = $2.55

Arbitrage Process

- Dealers can **strip a T-bond** into its individual cash flows **or combine the individual cash flows** into a bond
- If the bond is priced **less** than the arbitrage free value: Buy the bond, sell the pieces
- If the bond is priced **higher** than the arbitrage-free value: Buy the pieces, make a bond, sell the bond

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Fixed Income: Analysis and Valuation

- **Yield Measures, Spot Rates, and Forward Rates**

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LOS 66.a, CFAI Vol. 5 p. 536 Yield Measures, Spot Rates, Forward Rates

Sources of Bond Return

1. Coupon interest
2. Capital gain or loss when principal is repaid
3. Income from reinvestment of cash flows

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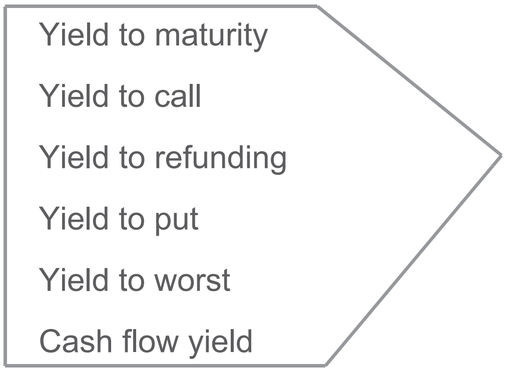
LOS 66.b, CFAI Vol. 5 p. 537 Yield Measures, Spot Rates, Forward Rates

Traditional Measures of Yield

Nominal yield (stated coupon rate)

Current yield



Yield to maturity

Yield to call

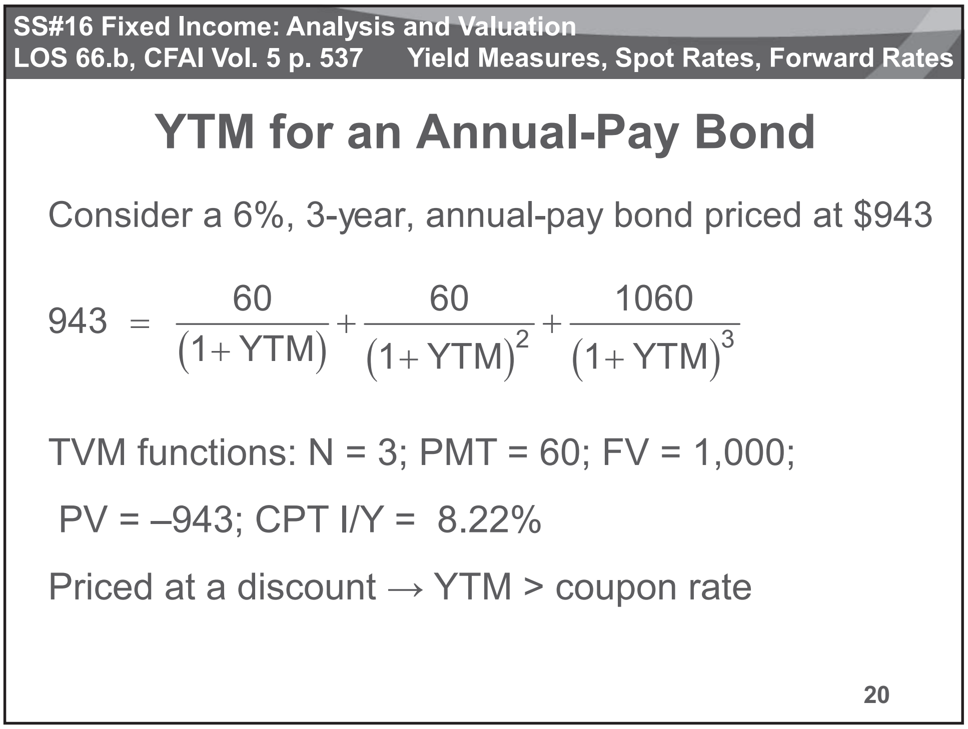
Yield to refunding

Yield to put

Yield to worst

Cash flow yield

IRR-based yields



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LOS 66.b, CFAI Vol. 5 p. 537 Yield Measures, Spot Rates, Forward Rates

YTM for an Annual-Pay Bond

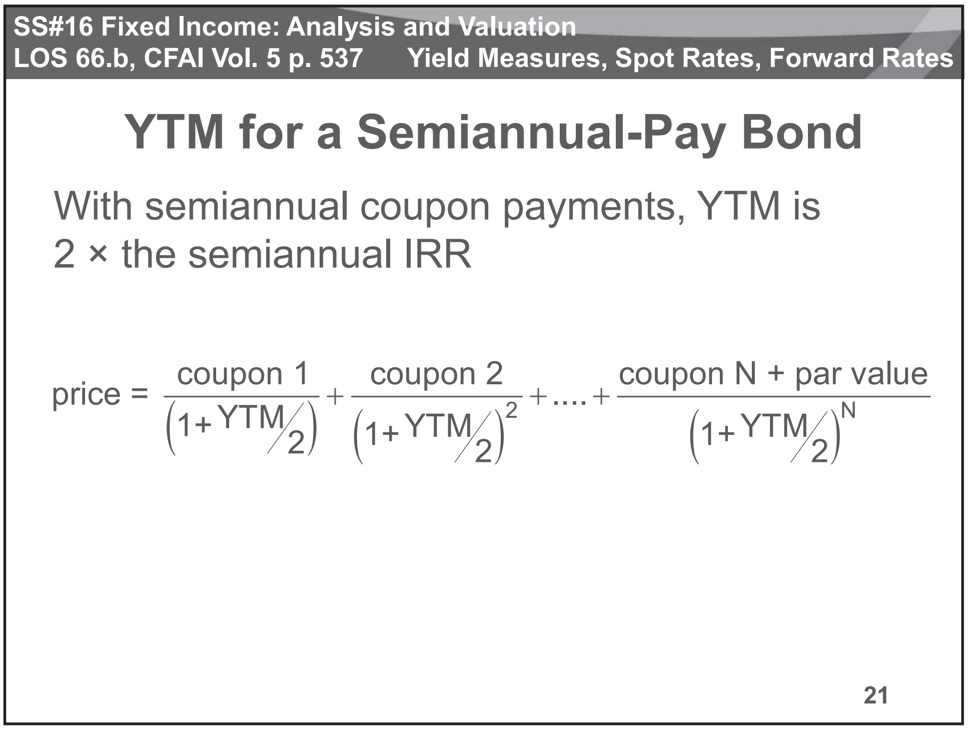
Consider a 6%, 3-year, annual-pay bond priced at $943

$$943 = \frac{60}{(1+\text{YTM})} + \frac{60}{(1+\text{YTM})^2} + \frac{1060}{(1+\text{YTM})^3}$$

TVM functions: N = 3; PMT = 60; FV = 1,000;

PV = –943; CPT I/Y = 8.22%

Priced at a discount → YTM > coupon rate



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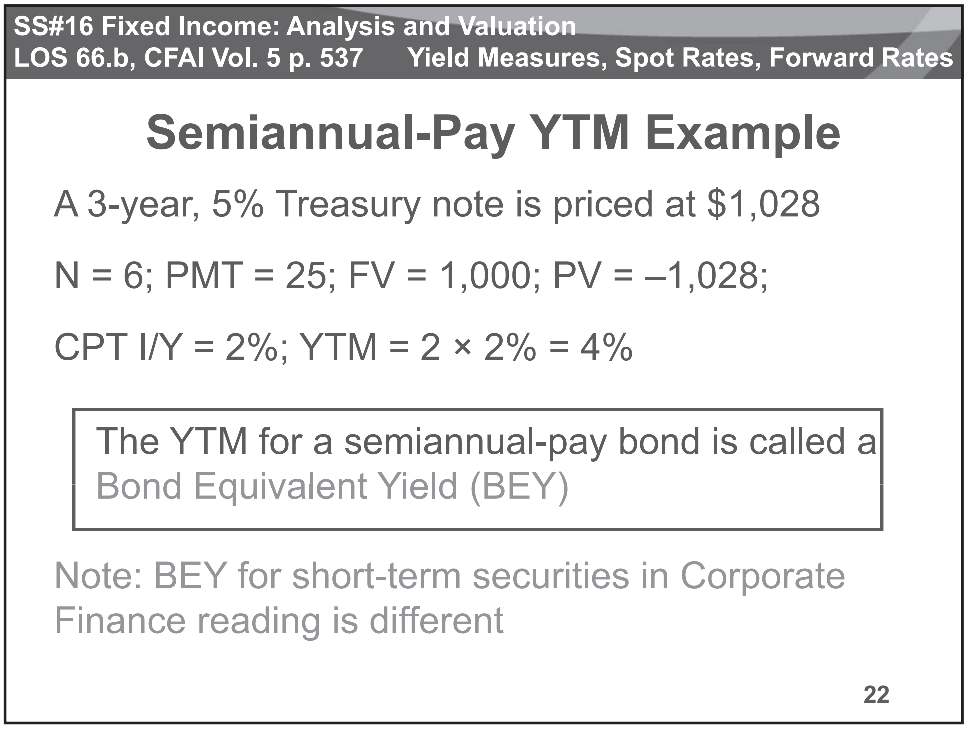
LOS 66.b, CFAI Vol. 5 p. 537 Yield Measures, Spot Rates, Forward Rates

YTM for a Semiannual-Pay Bond

With semiannual coupon payments, YTM is 2 × the semiannual IRR

$$\text{price} = \frac{\text{coupon 1}}{\left(1+\text{YTM}/2\right)} + \frac{\text{coupon 2}}{\left(1+\text{YTM}/2\right)^2} + \ldots. + \frac{\text{coupon N + par value}}{\left(1+\text{YTM}/2\right)^N}$$



Semiannual-Pay YTM Example

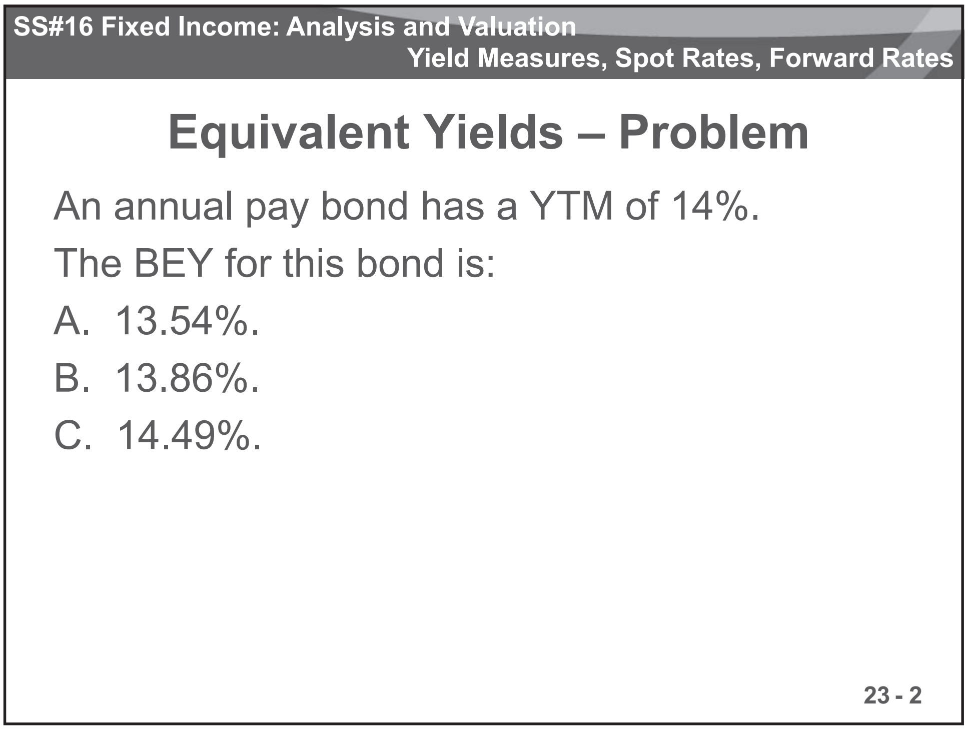
A 3-year, 5% Treasury note is priced at $1,028

N = 6; PMT = 25; FV = 1,000; PV = –1,028;

CPT I/Y = 2%; YTM = 2 × 2% = 4%

The YTM for a semiannual-pay bond is called a Bond Equivalent Yield (BEY)

Note: BEY for short-term securities in Corporate Finance reading is different



Equivalent Yields – Problem

An annual pay bond has a YTM of 14%.

The BEY for this bond is:

A. 13.54%.

B. 13.86%.

C. 14.49%.

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LOS 66.b, CFAI Vol. 5 p. 537 Yield Measures, Spot Rates, Forward Rates

Current Yield (Ignores Movement Toward Par Value)

$$\text{Current yield} = \frac{\text{Annual coupon payment}}{\text{Current price}}$$

For an 8%, 3-year (semiannual-pay) bond priced at 901.65

$$\text{Current yield} = \frac{80}{901.65} = 8.873\% \quad \text{YTM} = 12\%$$

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Yield Measures, Spot Rates, Forward Rates

Yield Measures – Problem

For a bond trading at a premium, order the coupon (nominal) yield, current yield, and YTM from smallest to largest.

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LOS 66.b, CFAI Vol. 5 p. 537 Yield Measures, Spot Rates, Forward Rates

Yield to First Call or Refunding

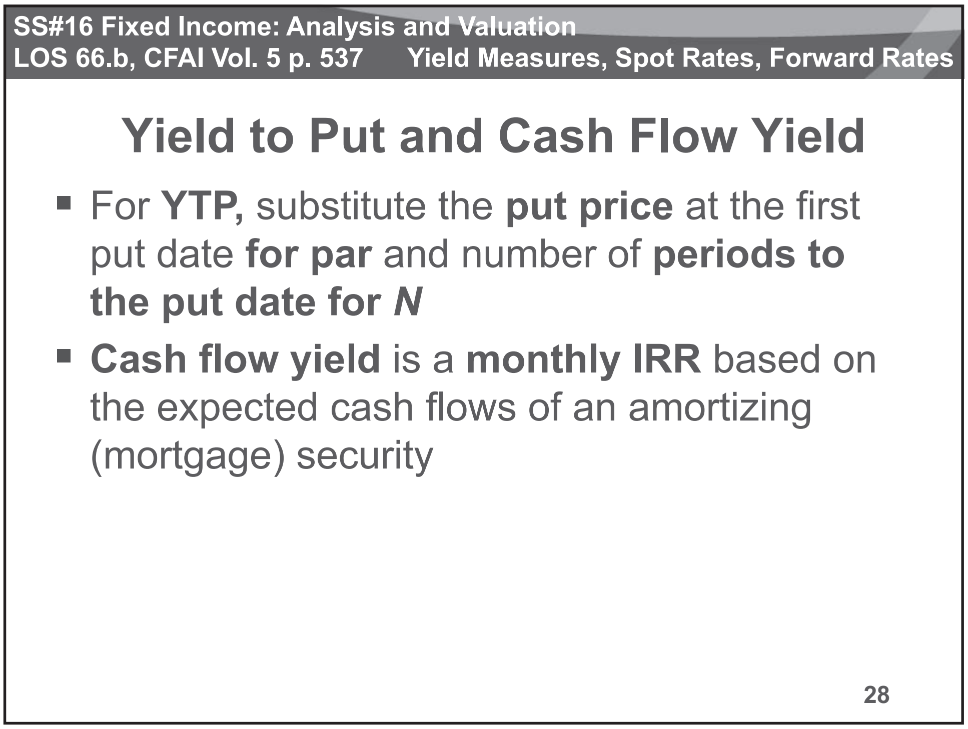
- For YTFC, substitute the call price at the first call date for par and number of periods to the first call date for *N*
- Use yield to refunding when bond is currently callable but has refunding protection
- Yield to worst is the lowest of YTM and the YTCs for all the call dates and prices

SS#16 Fixed Income: Analysis and Valuation
Yield Measures, Spot Rates, Forward Rates

Yield to Call – Problem

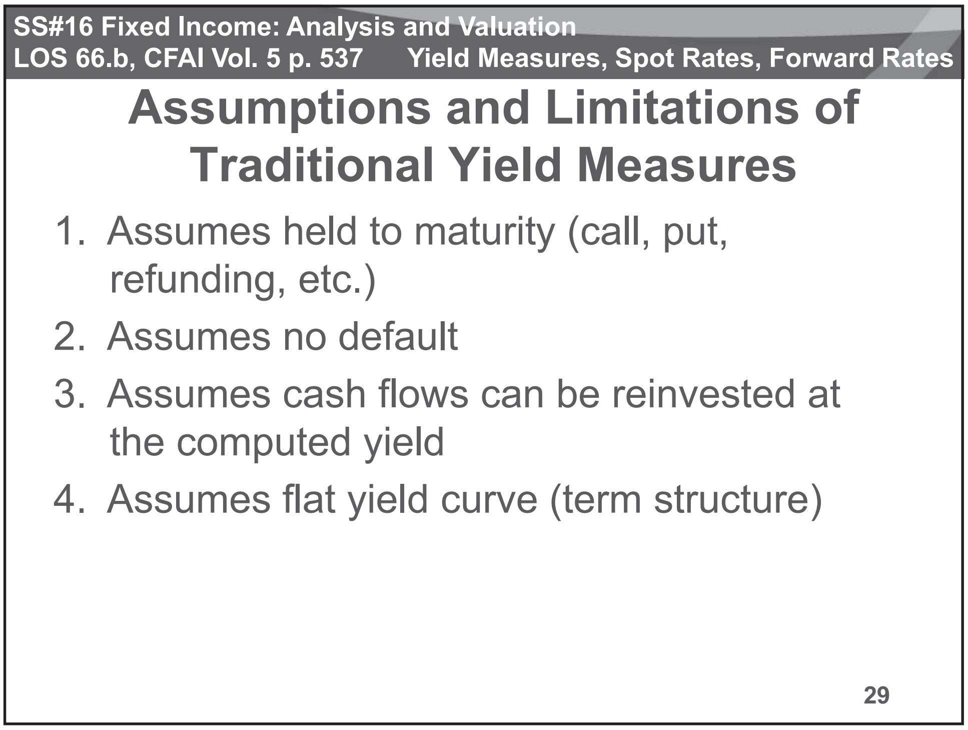
Consider a 10-year, 5% bond priced at $1,028
What is the YTM? ____________________

If it is callable in two years at 101, what is the YTC? ____________________



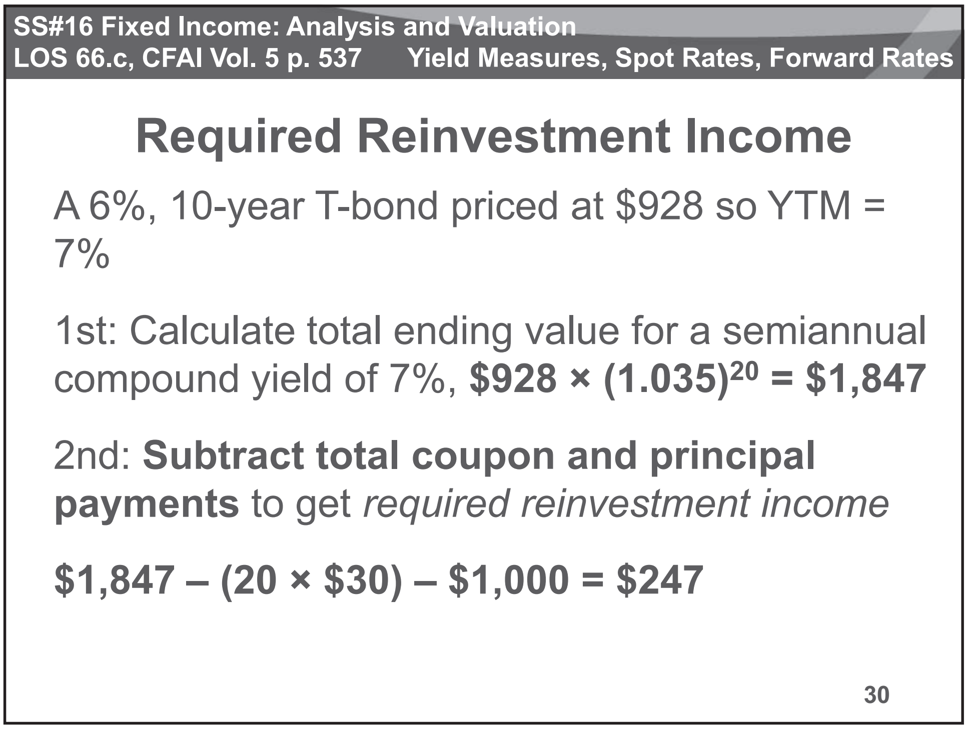
Yield to Put and Cash Flow Yield

- For **YTP,** substitute the **put price** at the first put date **for par** and number of **periods to the put date for *N***
- **Cash flow yield** is a **monthly IRR** based on the expected cash flows of an amortizing (mortgage) security



Assumptions and Limitations of Traditional Yield Measures

1. Assumes held to maturity (call, put, refunding, etc.)
2. Assumes no default
3. Assumes cash flows can be reinvested at the computed yield
4. Assumes flat yield curve (term structure)



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LOS 66.c, CFAI Vol. 5 p. 537 Yield Measures, Spot Rates, Forward Rates

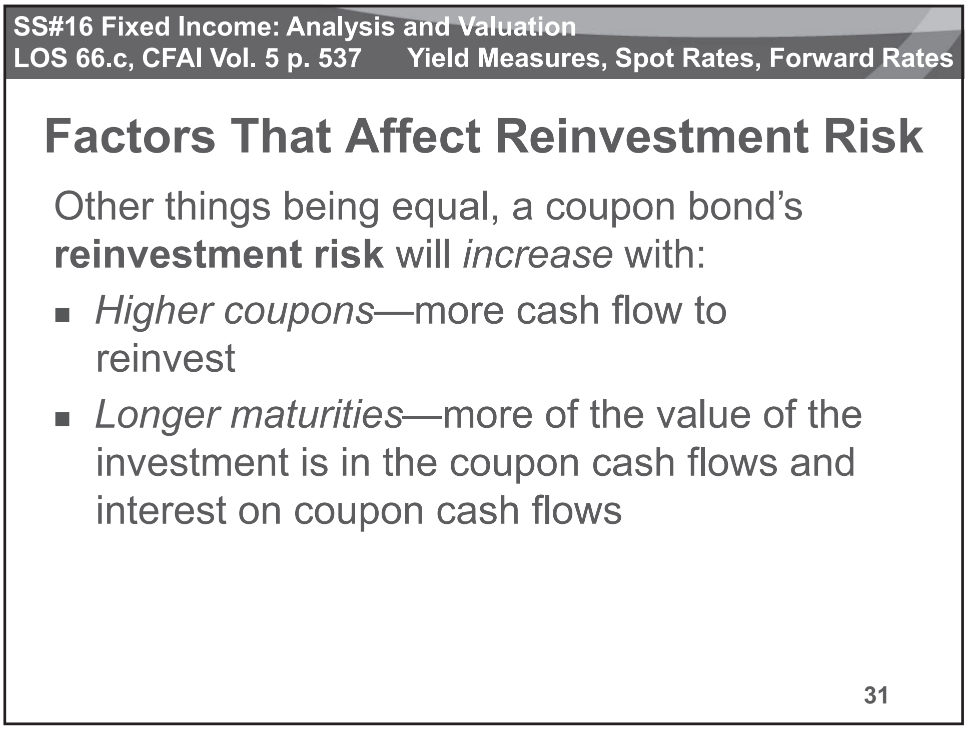
Required Reinvestment Income

A 6%, 10-year T-bond priced at $928 so YTM = 7%

1st: Calculate total ending value for a semiannual compound yield of 7%, **$\$928 \times (1.035)^{20} = \$1{,}847$**

2nd: **Subtract total coupon and principal payments** to get *required reinvestment income*

$\$1{,}847 – (20 \times \$30) – \$1{,}000 = \$247$



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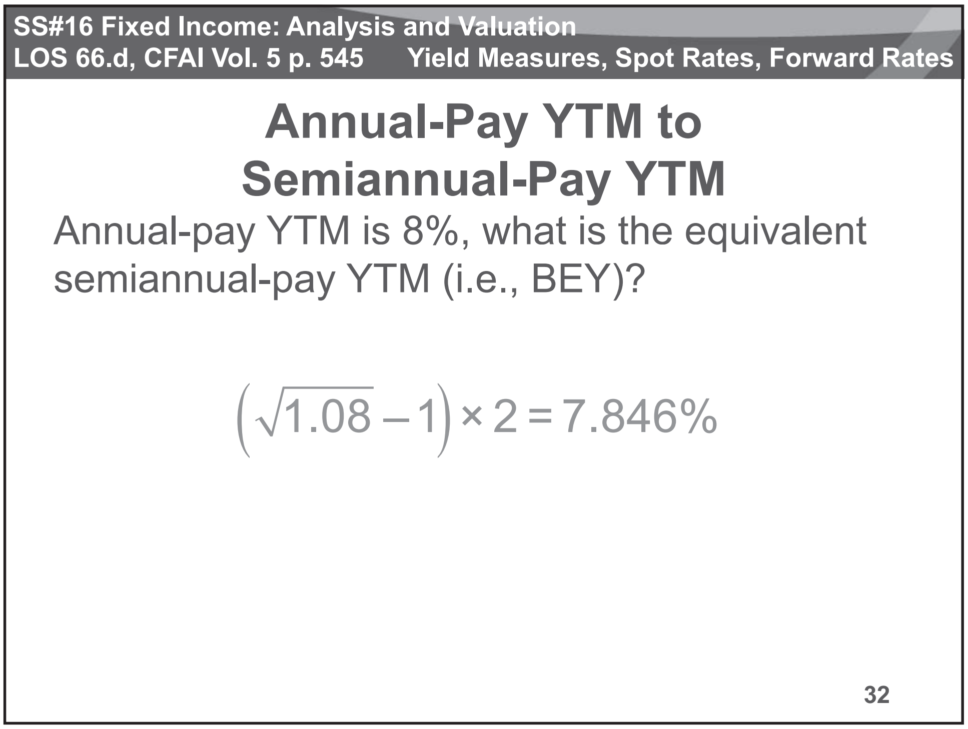
LOS 66.c, CFAI Vol. 5 p. 537 Yield Measures, Spot Rates, Forward Rates

Factors That Affect Reinvestment Risk

Other things being equal, a coupon bond's **reinvestment risk** will *increase* with:

- *Higher coupons*—more cash flow to reinvest
- *Longer maturities*—more of the value of the investment is in the coupon cash flows and interest on coupon cash flows



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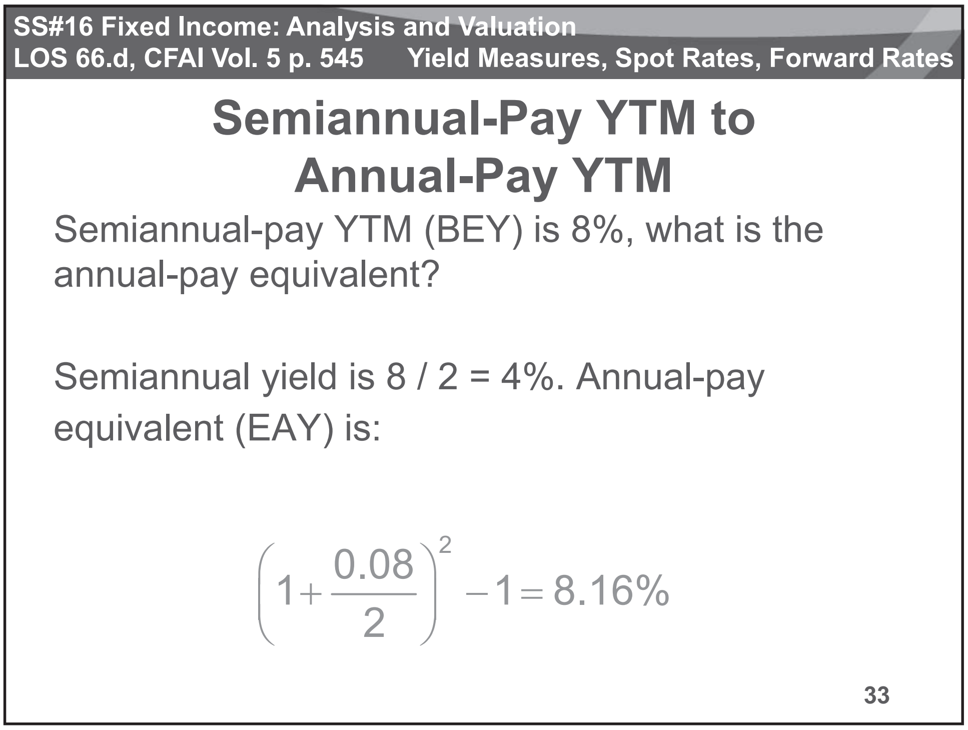
Annual-Pay YTM to Semiannual-Pay YTM

Annual-pay YTM is 8%, what is the equivalent semiannual-pay YTM (i.e., BEY)?

$$\left(\sqrt{1.08}-1\right)\times 2 = 7.846\%$$



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LOS 66.d, CFAI Vol. 5 p. 545 Yield Measures, Spot Rates, Forward Rates

Semiannual-Pay YTM to Annual-Pay YTM

Semiannual-pay YTM (BEY) is 8%, what is the annual-pay equivalent?

Semiannual yield is 8 / 2 = 4%. Annual-pay equivalent (EAY) is:

$$\left(1+\frac{0.08}{2}\right)^2-1=8.16\%$$

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LOS 66.e, CFAI Vol. 5 p. 554 Yield Measures, Spot Rates, Forward Rates

Theoretical Treasury Spot Rates

Begin with prices for 6-month, 1-year, and 18-month Treasuries:

6-month T-bill price is 98.30, 6-month discount rate is 1.73% $\frac{1{,}000}{1.0173} = 983$ BEY = 2 × 1.73 = 3.46%

1-year 4% T-note is priced at 99.50

$$\frac{20}{1.0173} + \frac{1{,}020}{(1+?)^2} = 995 \quad 995 - \frac{20}{1.0173} = 975.34 = \frac{1{,}020}{(1+?)^2}$$

$$? = \sqrt{\frac{1{,}020}{975.34}} - 1 = 2.26\%, \quad \text{BEY} = 2 \times 2.26 = 4.52\%$$

SS#16 Fixed Income: Analysis and Valuation
LOS 66.e, CFAI Vol. 5 p. 554 Yield Measures, Spot Rates, Forward Rates

Theoretical Treasury Spot Rates

Begin with prices for 6-month, 1-year, and 18-month Treasuries:

1.5-year 4.5% T-note is priced at 98.60

$$\frac{22.5}{1.0173} + \frac{22.5}{(1.0226)^2} + \frac{1{,}022.5}{(1+?)^3} = 986 \quad 986 - \frac{22.5}{1.0173} - \frac{22.5}{(1.0226)^2} = 942.37 = \frac{1{,}022.5}{(1+?)^3}$$

$$? = \sqrt[3]{\frac{1{,}022.5}{942.37}} - 1 = 2.76\%, \quad \text{BEY} = 2 \times 2.76 = 5.52\%$$

By “bootstrapping,” we calculated the 1-year spot rate = 4.52% and the 1.5-year spot rate = 5.52%

SS#16 Fixed Income: Analysis and Valuation
LOS 66.e, CFAI Vol. 5 p. 554 **Yield Measures, Spot Rates, Forward Rates**

Valuing a Bond With Spot Rates

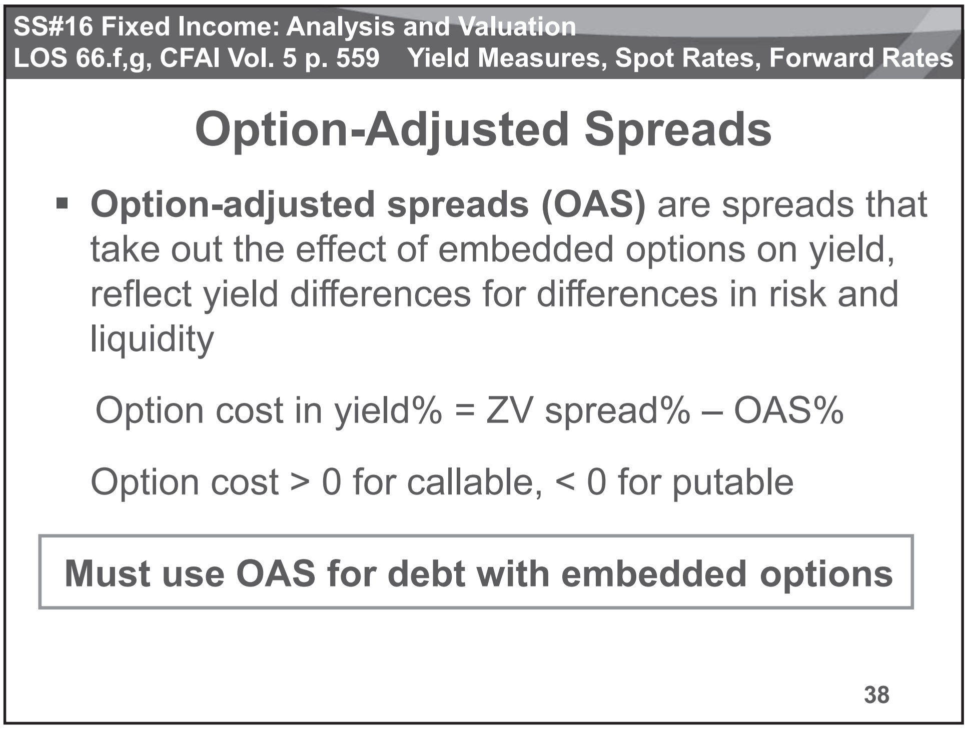
Use the spot rates we calculated to value a 5% 18-month Treasury note.

$$\frac{25}{1.0173} + \frac{25}{(1.0226)^2} + \frac{1{,}025}{(1.0276)^3} = 993.09$$

SS#16 Fixed Income: Analysis and Valuation
LOS 66.f, CFAI Vol. 5 p. 559 **Yield Measures, Spot Rates, Forward Rates**

Nominal and Zero-Volatility Spreads

- **Nominal spreads** are just differences in YTMs
- **Zero-volatility (ZV) spreads** are the (parallel) spread to Treasury spot-rate curve to get PV = market price
- Equal amounts added to each spot rate to get PV = market price



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LOS 66.f,g, CFAI Vol. 5 p. 559 Yield Measures, Spot Rates, Forward Rates

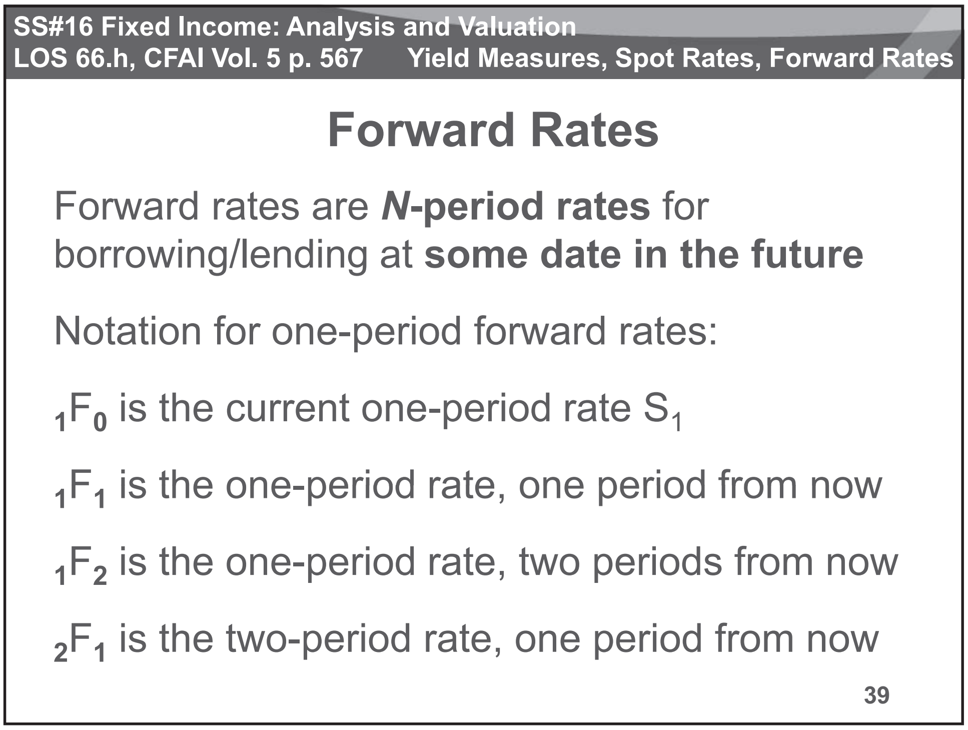
Option-Adjusted Spreads

- **Option-adjusted spreads (OAS)** are spreads that take out the effect of embedded options on yield, reflect yield differences for differences in risk and liquidity

Option cost in yield% = ZV spread% – OAS%

Option cost > 0 for callable, < 0 for putable

Must use OAS for debt with embedded options



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LOS 66.h, CFAI Vol. 5 p. 567 Yield Measures, Spot Rates, Forward Rates

Forward Rates

Forward rates are ***N*-period rates** for borrowing/lending at **some date in the future**

Notation for one-period forward rates:

${}_1F_0$ is the current one-period rate S_1

${}_1F_1$ is the one-period rate, one period from now

${}_1F_2$ is the one-period rate, two periods from now

${}_2F_1$ is the two-period rate, one period from now

Spot Rates and Forward Rates

$$(1+S_3)^3 = (1+S_1)(1+{}_1F_1)(1+{}_1F_2)$$

$$(1+S_3)^3 = (1+S_1)(1+{}_2F_1)^2$$

$$(1+S_3)^3 = (1+S_2)^2(1+{}_1F_2)$$

Cost of borrowing for 3 year at S_3 should equal cost of:

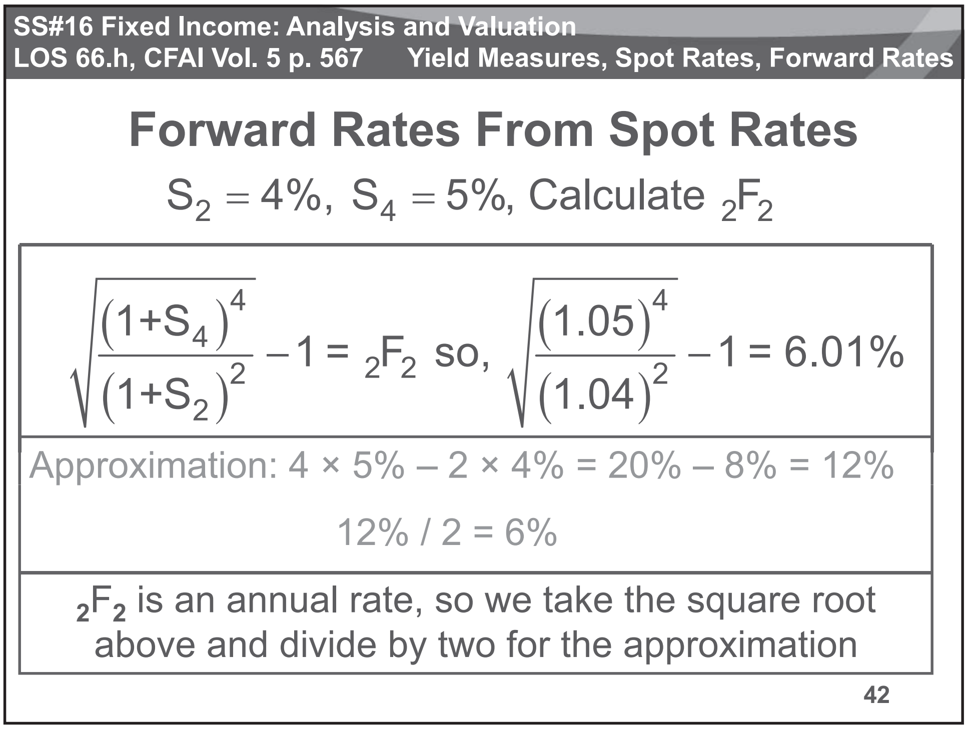
- Borrowing for 1 year at S_1, 1 year at ${}_1F_{1,}$ and 1 year at ${}_1F_2$
- Borrowing for 1 year at S_1 and for 2 years at ${}_2F_1$
- Borrowing for 2 years at S_2 and for 1 year at ${}_1F_2$

Forward Rates From Spot Rates

$S_2 = 4\%, S_3 = 5\%$, calculate ${}_1F_2$

$$\frac{(1+S_3)^3}{(1+S_2)^2} - 1 = {}_1F_2 \text{ so, } \frac{(1.05)^3}{(1.04)^2} - 1 = 7.03\%$$

Approximation: 3 × 5% – 2 × 4% = 15% – 8% = 7%



Forward Rates From Spot Rates

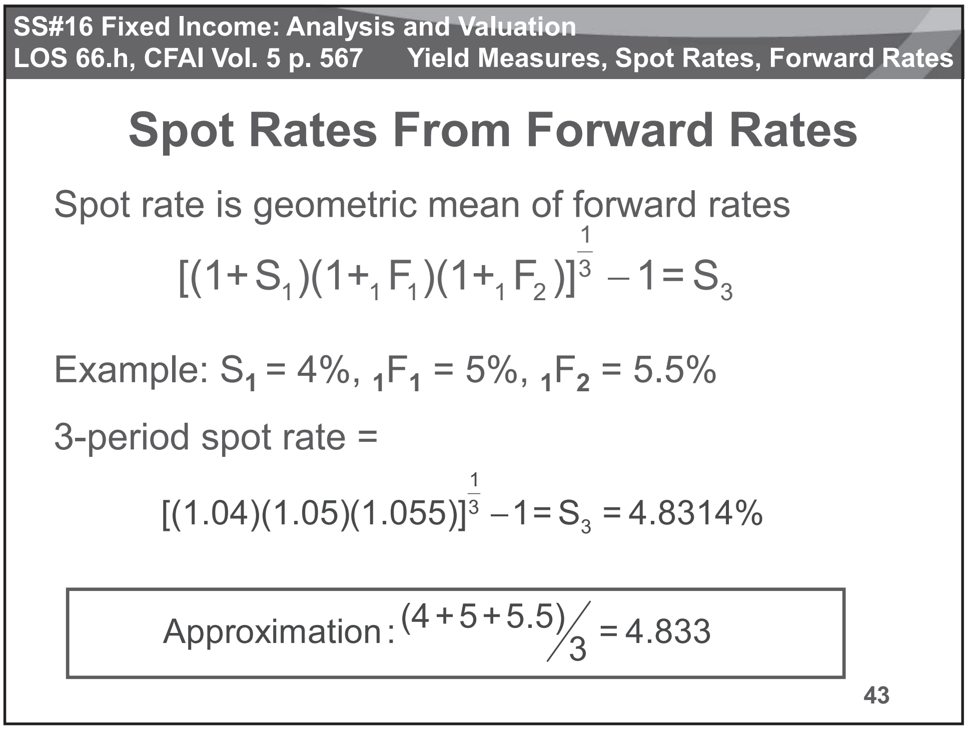
$S_2 = 4\%,\ S_4 = 5\%$, Calculate ${}_2F_2$

$$\sqrt{\frac{(1+S_4)^4}{(1+S_2)^2}} - 1 = {}_2F_2 \text{ so, } \sqrt{\frac{(1.05)^4}{(1.04)^2}} - 1 = 6.01\%$$

Approximation: 4 × 5% – 2 × 4% = 20% – 8% = 12%

12% / 2 = 6%

${}_2F_2$ is an annual rate, so we take the square root above and divide by two for the approximation



Spot Rates From Forward Rates

Spot rate is geometric mean of forward rates

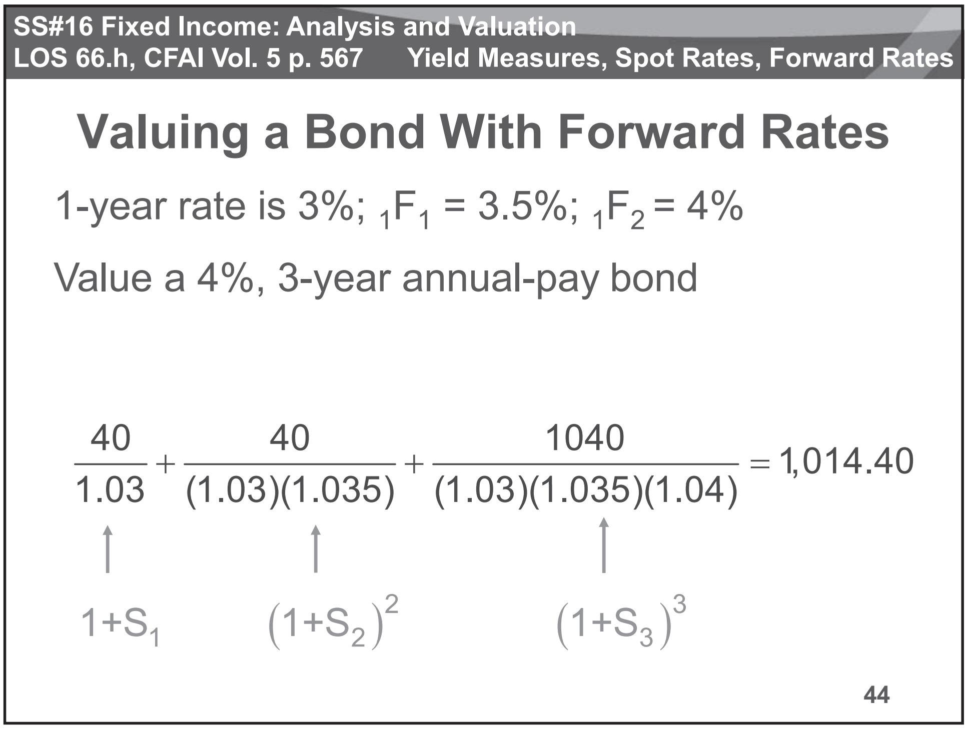
$$[(1+S_1)(1+{}_1F_1)(1+{}_1F_2)]^{\frac{1}{3}} - 1 = S_3$$

Example: $S_1 = 4\%$, ${}_1F_1 = 5\%$, ${}_1F_2 = 5.5\%$

3-period spot rate =

$$[(1.04)(1.05)(1.055)]^{\frac{1}{3}} - 1 = S_3 = 4.8314\%$$

$$\text{Approximation}: (4+5+5.5)/3 = 4.833$$



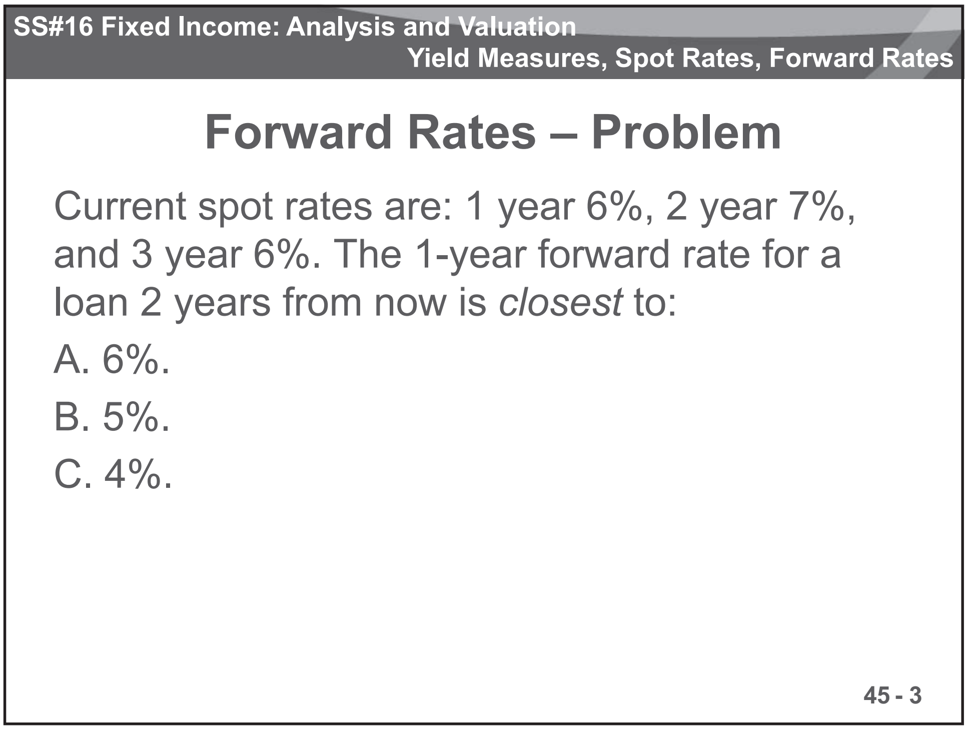
SS#16 Fixed Income: Analysis and Valuation
LOS 66.h, CFAI Vol. 5 p. 567 Yield Measures, Spot Rates, Forward Rates

Valuing a Bond With Forward Rates

1-year rate is 3%; ${}_1F_1 = 3.5\%$; ${}_1F_2 = 4\%$

Value a 4%, 3-year annual-pay bond

$$\frac{40}{\underset{\underset{1+S_1}{\uparrow}}{1.03}}+\frac{40}{\underset{\underset{(1+S_2)^2}{\uparrow}}{(1.03)(1.035)}}+\frac{1040}{\underset{\underset{(1+S_3)^3}{\uparrow}}{(1.03)(1.035)(1.04)}}=1{,}014.40$$



SS#16 Fixed Income: Analysis and Valuation
Yield Measures, Spot Rates, Forward Rates

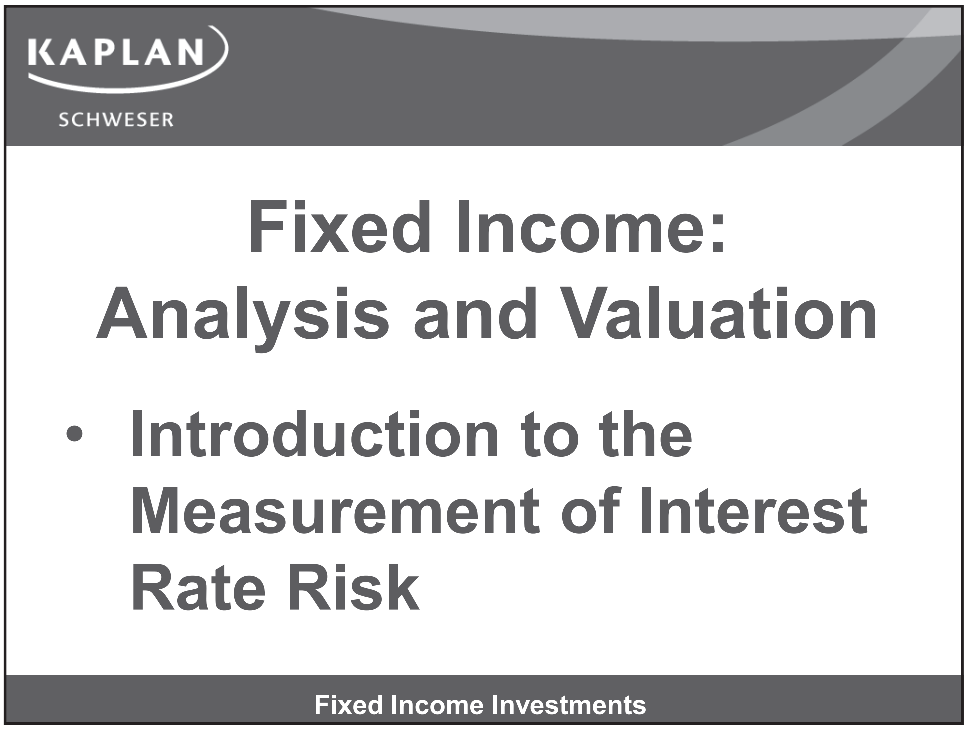
Forward Rates – Problem

Current spot rates are: 1 year 6%, 2 year 7%, and 3 year 6%. The 1-year forward rate for a loan 2 years from now is *closest* to:

A. 6%.

B. 5%.

C. 4%.

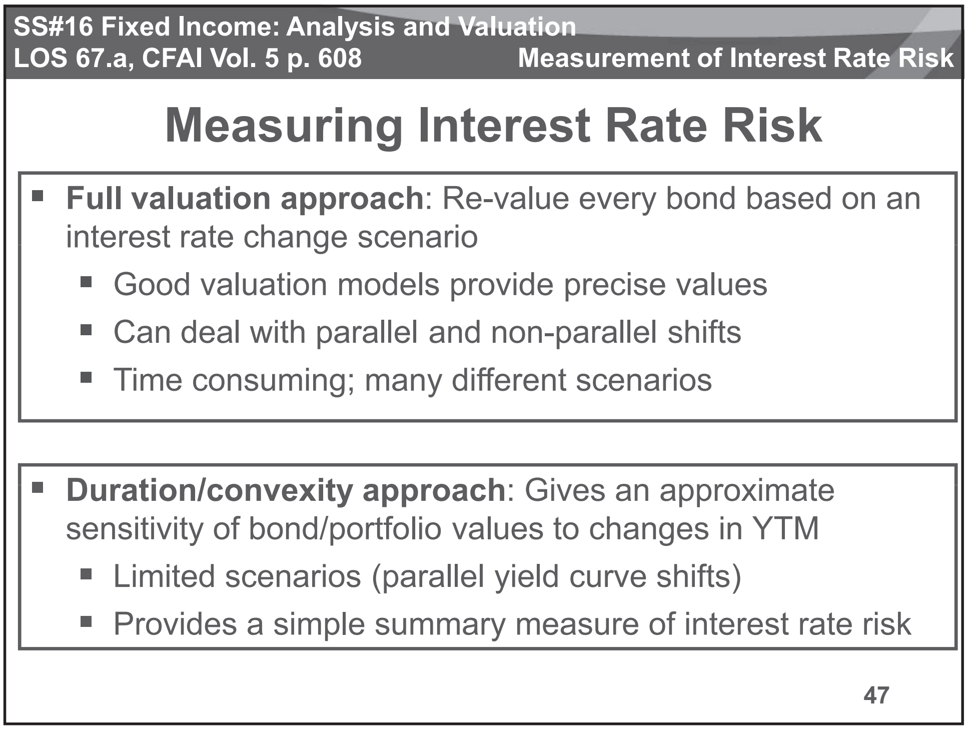


KAPLAN SCHWESER

Fixed Income: Analysis and Valuation

- **Introduction to the Measurement of Interest Rate Risk**

Fixed Income Investments

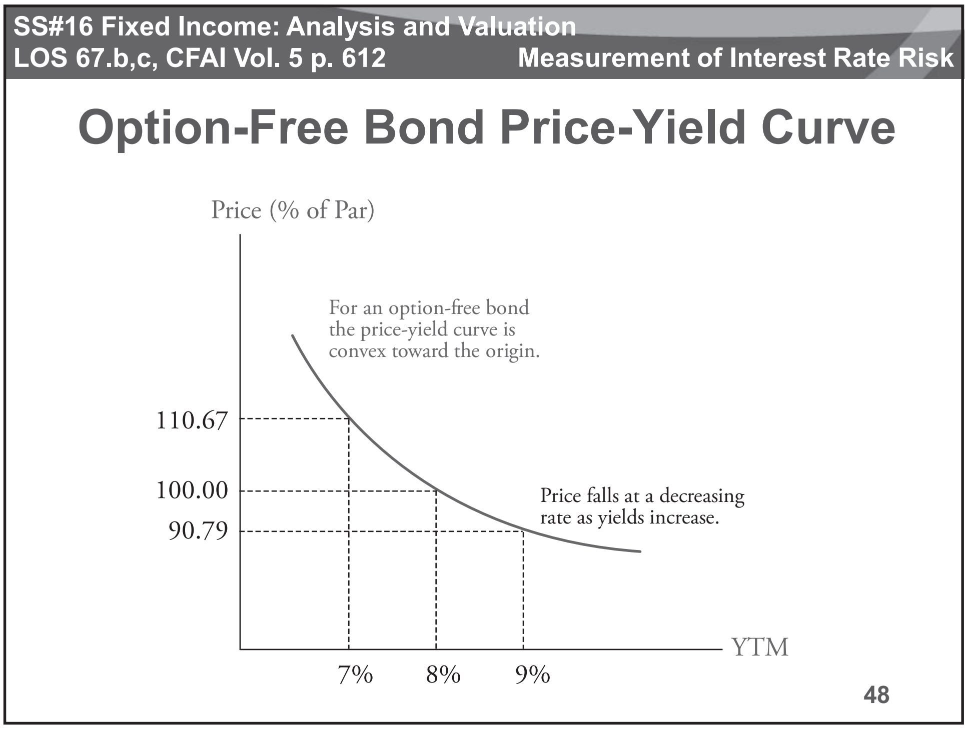


SS#16 Fixed Income: Analysis and Valuation
LOS 67.a, CFAI Vol. 5 p. 608 **Measurement of Interest Rate Risk**

Measuring Interest Rate Risk

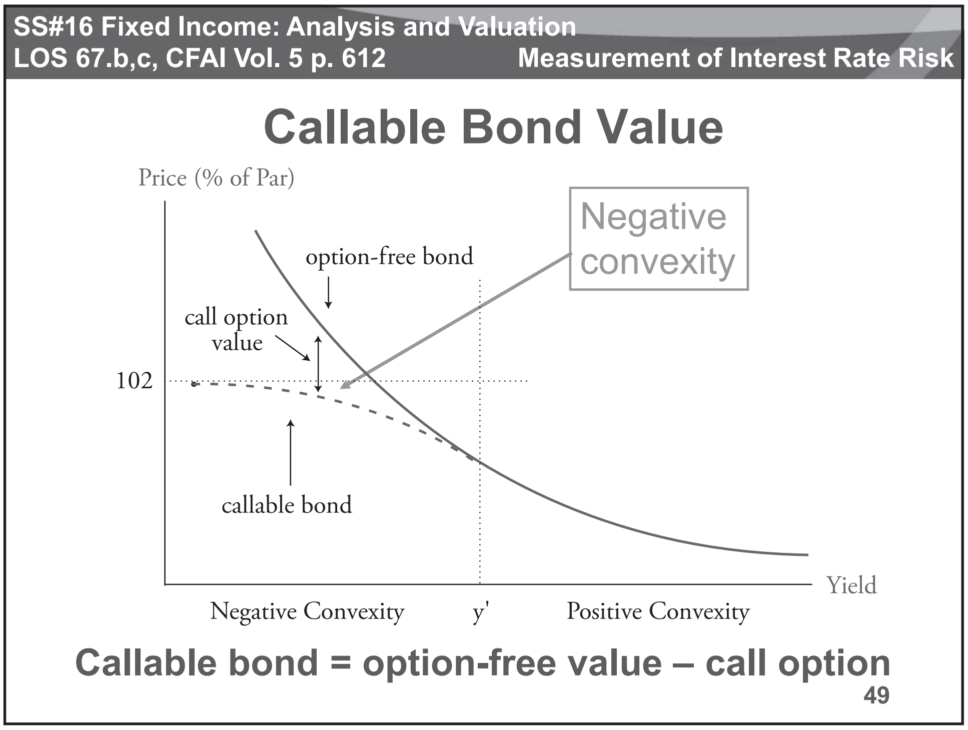
- **Full valuation approach**: Re-value every bond based on an interest rate change scenario
  - Good valuation models provide precise values
  - Can deal with parallel and non-parallel shifts
  - Time consuming; many different scenarios

- **Duration/convexity approach**: Gives an approximate sensitivity of bond/portfolio values to changes in YTM
  - Limited scenarios (parallel yield curve shifts)
  - Provides a simple summary measure of interest rate risk



SS#16 Fixed Income: Analysis and Valuation
LOS 67.b,c, CFAI Vol. 5 p. 612 **Measurement of Interest Rate Risk**

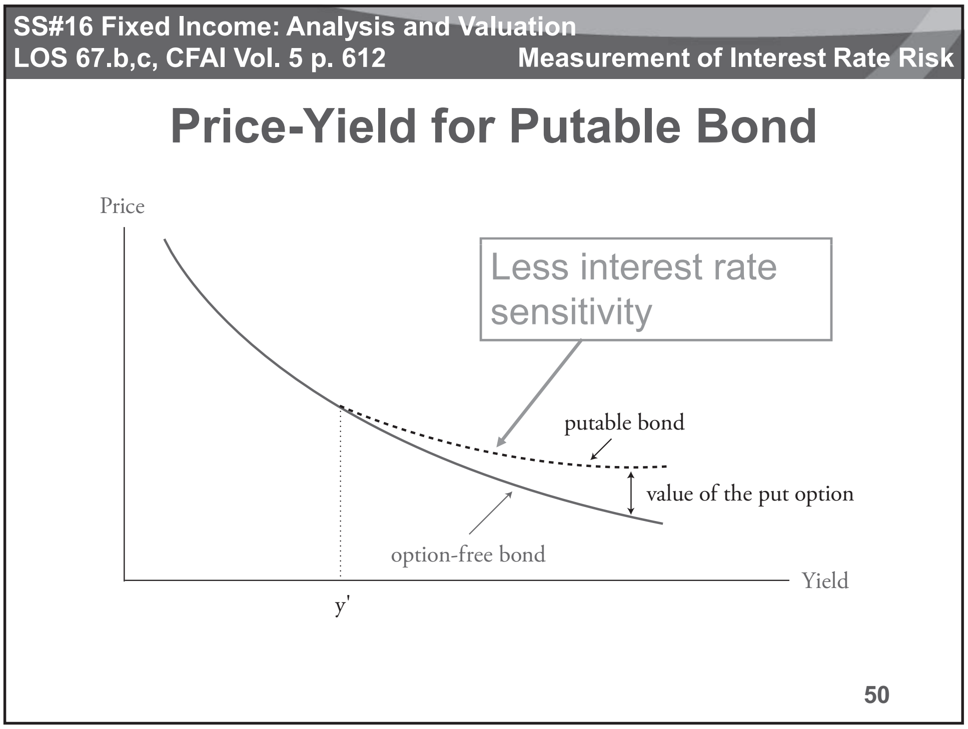
Option-Free Bond Price-Yield Curve



SS#16 Fixed Income: Analysis and Valuation
LOS 67.b,c, CFAI Vol. 5 p. 612 **Measurement of Interest Rate Risk**

Callable Bond Value

Callable bond = option-free value – call option



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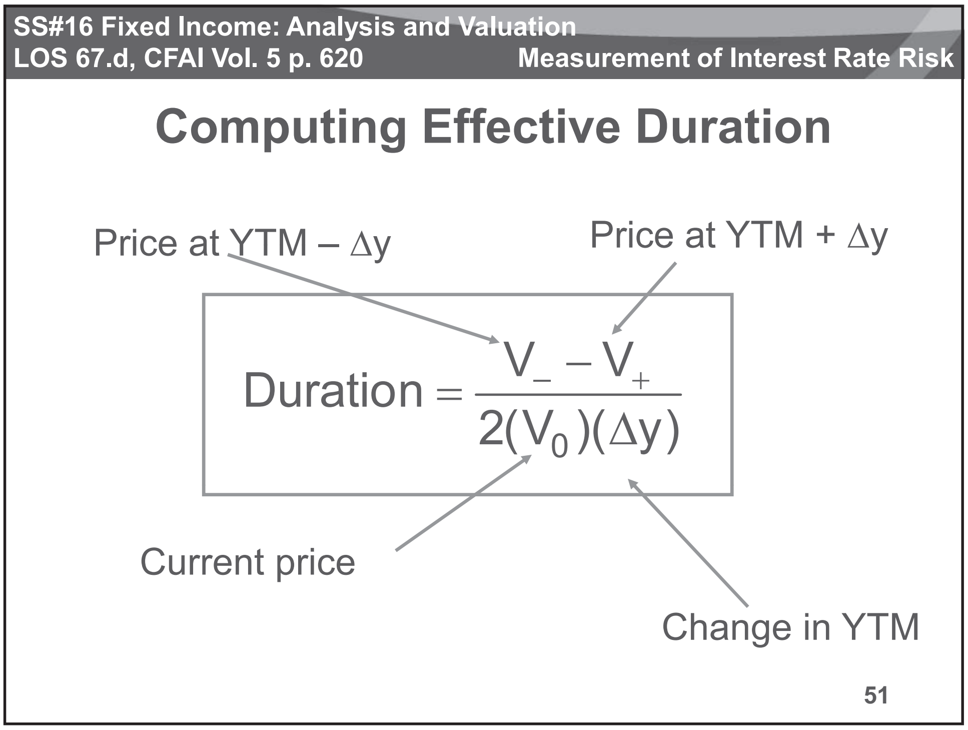
LOS 67.b,c, CFAI Vol. 5 p. 612 **Measurement of Interest Rate Risk**

Price-Yield for Putable Bond



SS#16 Fixed Income: Analysis and Valuation
LOS 67.d, CFAI Vol. 5 p. 620 **Measurement of Interest Rate Risk**

Computing Effective Duration

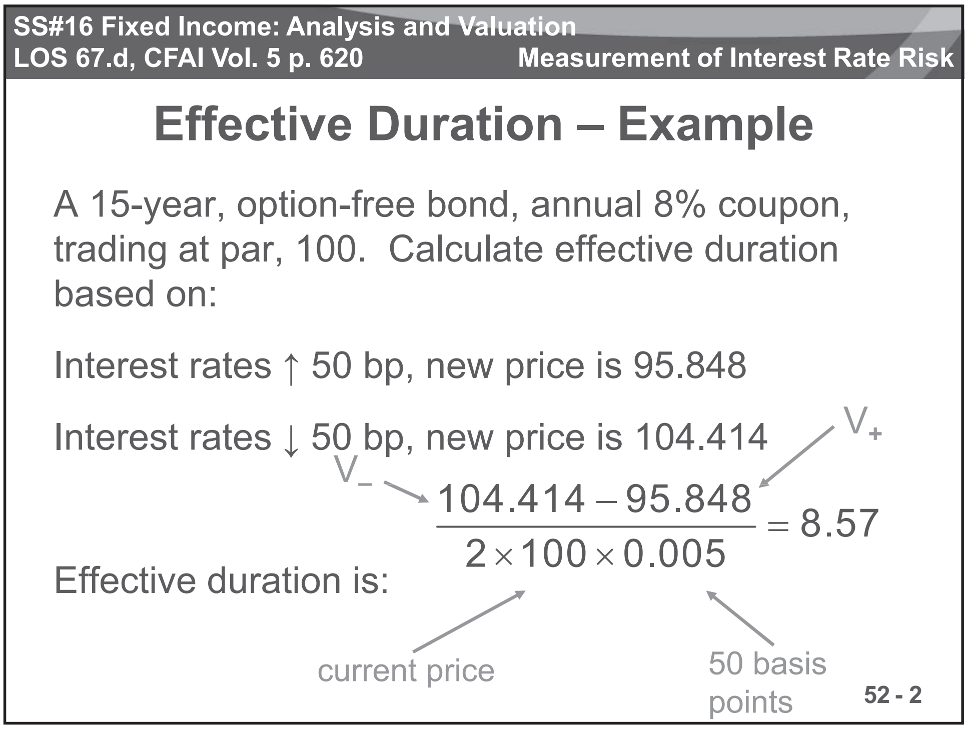
Price at YTM – Δy

Price at YTM + Δy

$$\boxed{\text{Duration} = \frac{V_{-} - V_{+}}{2(V_0)(\Delta y)}}$$

Current price

Change in YTM



SS#16 Fixed Income: Analysis and Valuation
LOS 67.d, CFAI Vol. 5 p. 620 **Measurement of Interest Rate Risk**

Effective Duration – Example

A 15-year, option-free bond, annual 8% coupon, trading at par, 100. Calculate effective duration based on:

Interest rates ↑ 50 bp, new price is 95.848

Interest rates ↓ 50 bp, new price is 104.414

V_+

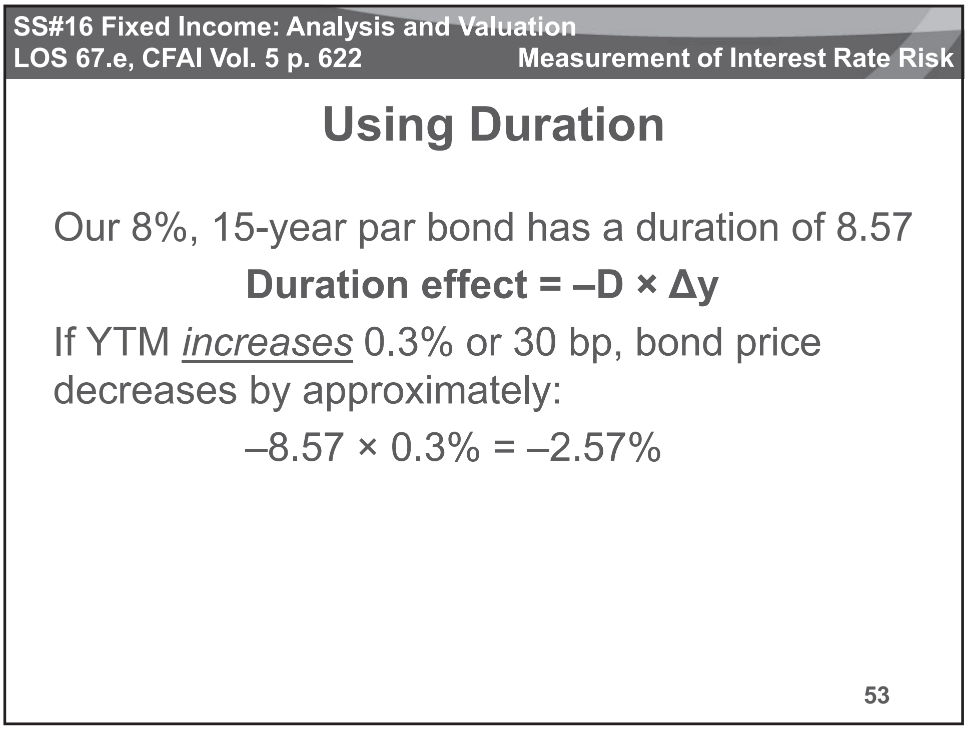
V_-

Effective duration is:

$$\frac{104.414 - 95.848}{2 \times 100 \times 0.005} = 8.57$$

current price

50 basis points



SS#16 Fixed Income: Analysis and Valuation
LOS 67.e, CFAI Vol. 5 p. 622 **Measurement of Interest Rate Risk**

Using Duration

Our 8%, 15-year par bond has a duration of 8.57

Duration effect = –D × Δy

If YTM *increases* 0.3% or 30 bp, bond price decreases by approximately:

–8.57 × 0.3% = –2.57%

SS#16 Fixed Income: Analysis and Valuation
LOS 67.f, CFAI Vol. 5 p. 628 **Measurement of Interest Rate Risk**

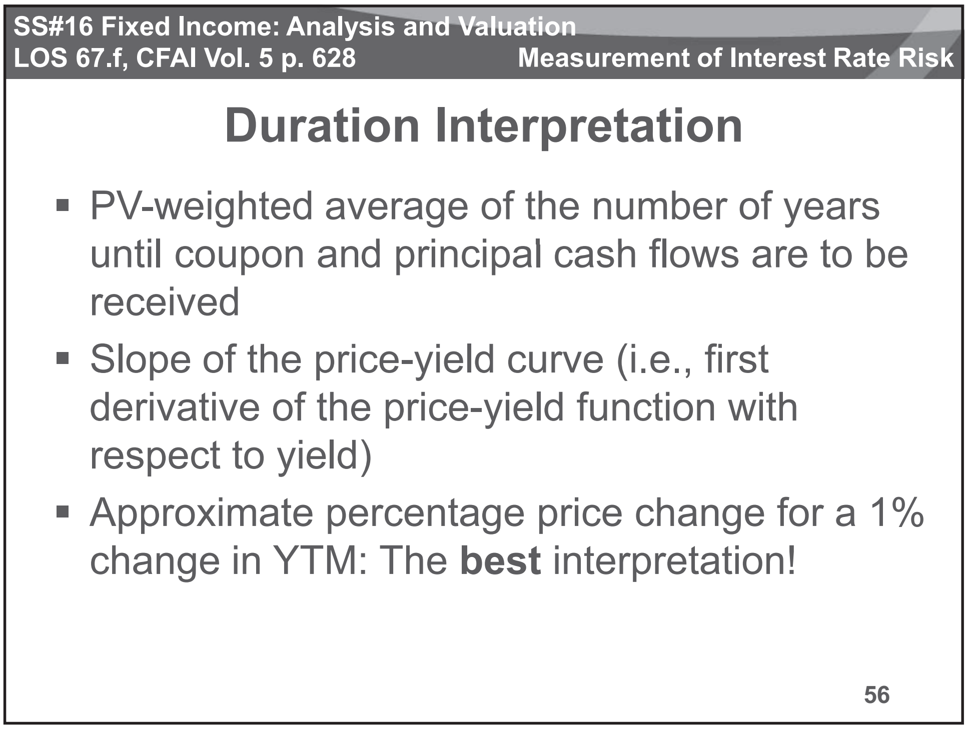
Duration Measures

- **Macaulay duration** is in years
  - Duration of a 5-year, zero-coupon bond is 5
  - 1% change in yield, 5% change in price
- **Modified duration** adjusts Macaulay duration for market yield, yield up → duration down
- **Effective duration** allows for cash flow changes as yield changes, must be used for bonds with embedded options

SS#16 Fixed Income: Analysis and Valuation
LOS 67.f, CFAI Vol. 5 p. 628 **Measurement of Interest Rate Risk**

Effective Duration

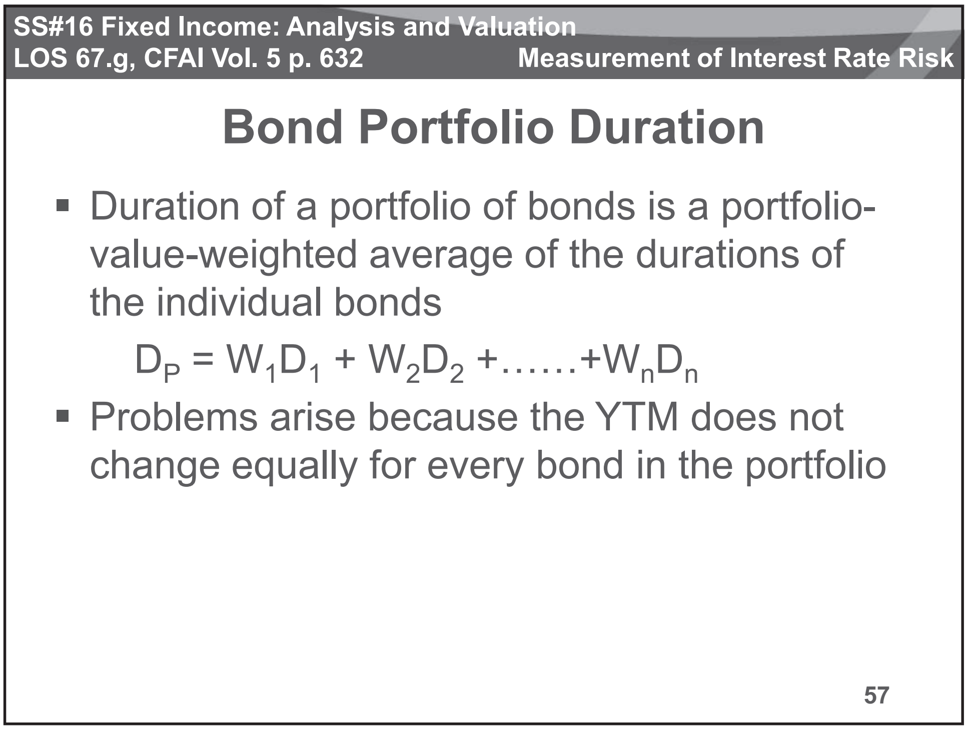
- Both Macaulay duration and modified duration are based on the promised cash flows and ignore call, put, and prepayment options
- **Effective duration** can be calculated using prices from a valuation model that includes the effects of **embedded options** (e.g., call feature)
- For option-free bonds, effective duration is very close to modified duration
- For bonds with **embedded options**, effective duration **must** be used



SS#16 Fixed Income: Analysis and Valuation
LOS 67.f, CFAI Vol. 5 p. 628 **Measurement of Interest Rate Risk**

Duration Interpretation

- PV-weighted average of the number of years until coupon and principal cash flows are to be received
- Slope of the price-yield curve (i.e., first derivative of the price-yield function with respect to yield)
- Approximate percentage price change for a 1% change in YTM: The **best** interpretation!



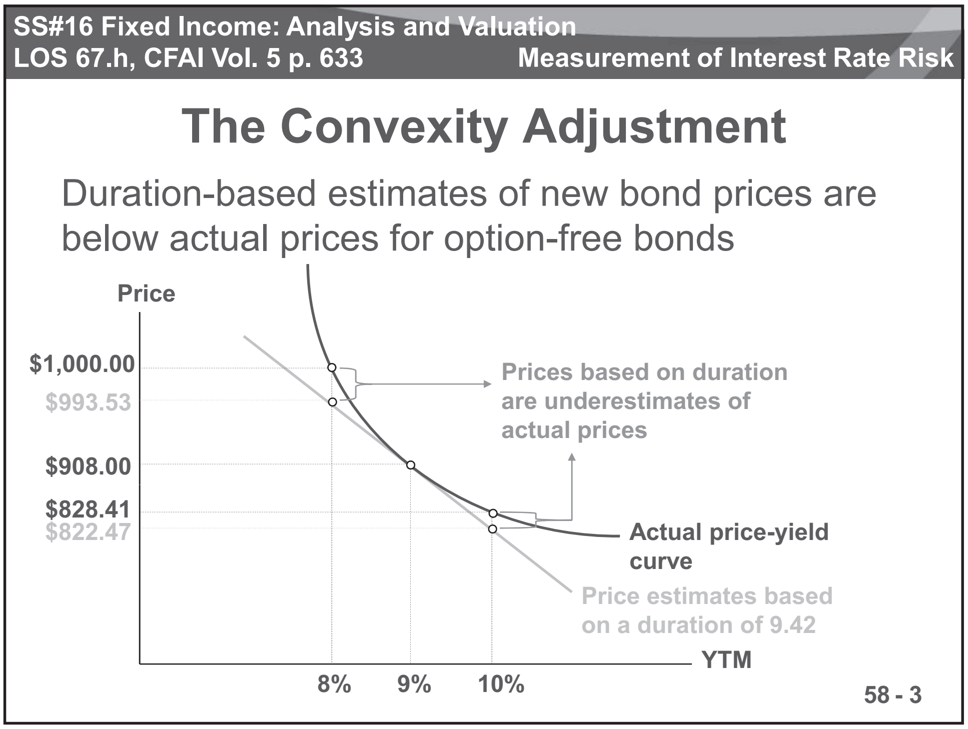
SS#16 Fixed Income: Analysis and Valuation
LOS 67.g, CFAI Vol. 5 p. 632 **Measurement of Interest Rate Risk**

Bond Portfolio Duration

- Duration of a portfolio of bonds is a portfolio-value-weighted average of the durations of the individual bonds

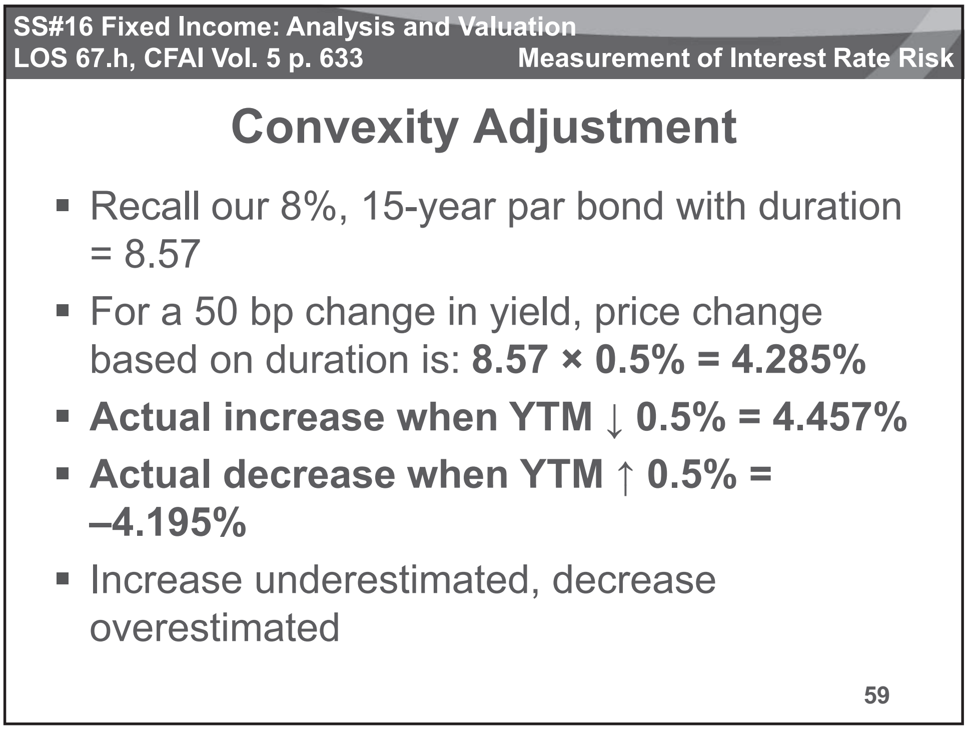
  $$D_P = W_1D_1 + W_2D_2 + \ldots\ldots + W_nD_n$$

- Problems arise because the YTM does not change equally for every bond in the portfolio



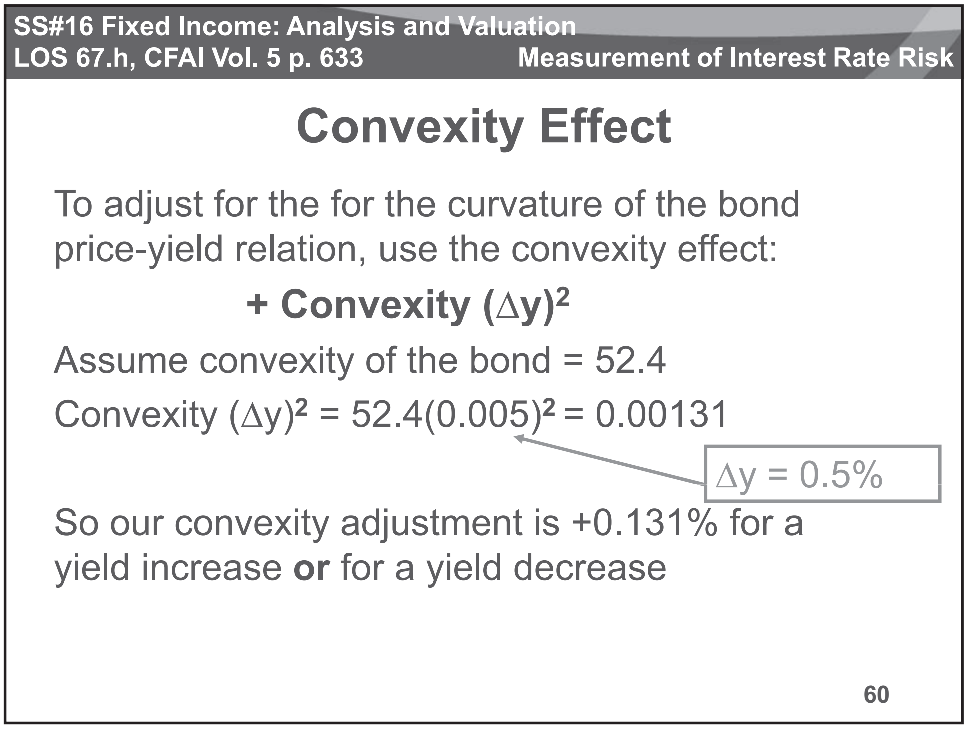
The Convexity Adjustment

Duration-based estimates of new bond prices are below actual prices for option-free bonds



Convexity Adjustment

- Recall our 8%, 15-year par bond with duration = 8.57
- For a 50 bp change in yield, price change based on duration is: **8.57 × 0.5% = 4.285%**
- **Actual increase when YTM ↓ 0.5% = 4.457%**
- **Actual decrease when YTM ↑ 0.5% = –4.195%**
- Increase underestimated, decrease overestimated



SS#16 Fixed Income: Analysis and Valuation
LOS 67.h, CFAI Vol. 5 p. 633 **Measurement of Interest Rate Risk**

Convexity Effect

To adjust for the for the curvature of the bond price-yield relation, use the convexity effect:

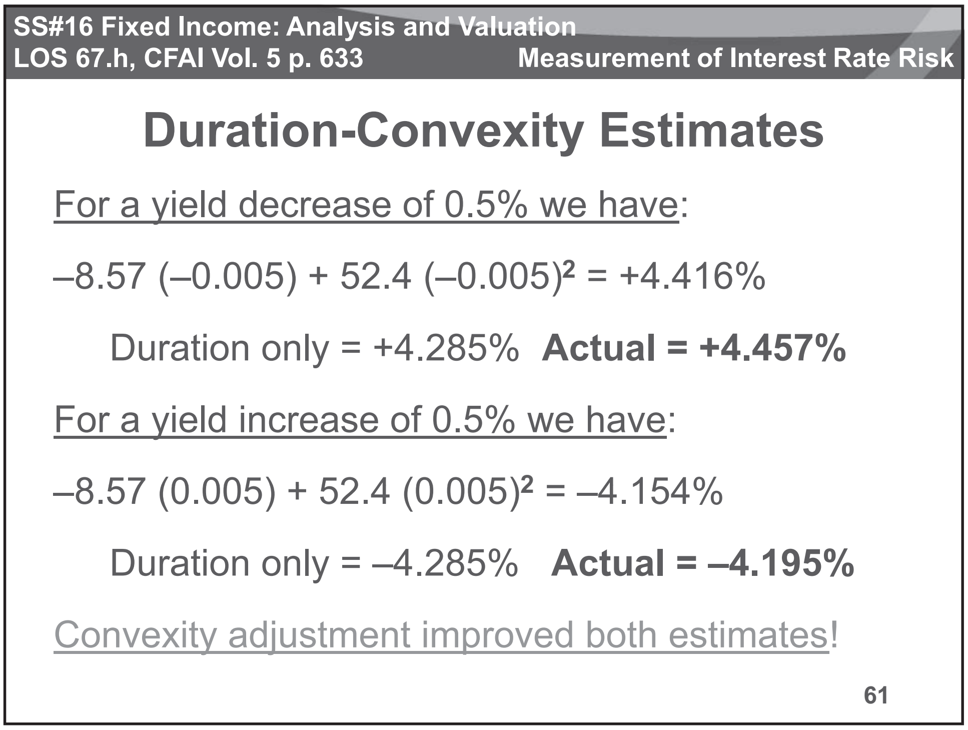
$$\mathbf{+\ Convexity\ (\Delta y)^2}$$

Assume convexity of the bond = 52.4

Convexity $(\Delta y)^2 = 52.4(0.005)^2 = 0.00131$

$\Delta y = 0.5\%$

So our convexity adjustment is +0.131% for a yield increase **or** for a yield decrease



SS#16 Fixed Income: Analysis and Valuation
LOS 67.h, CFAI Vol. 5 p. 633 **Measurement of Interest Rate Risk**

Duration-Convexity Estimates

<u>For a yield decrease of 0.5% we have</u>:

$-8.57\ (-0.005) + 52.4\ (-0.005)^2 = +4.416\%$

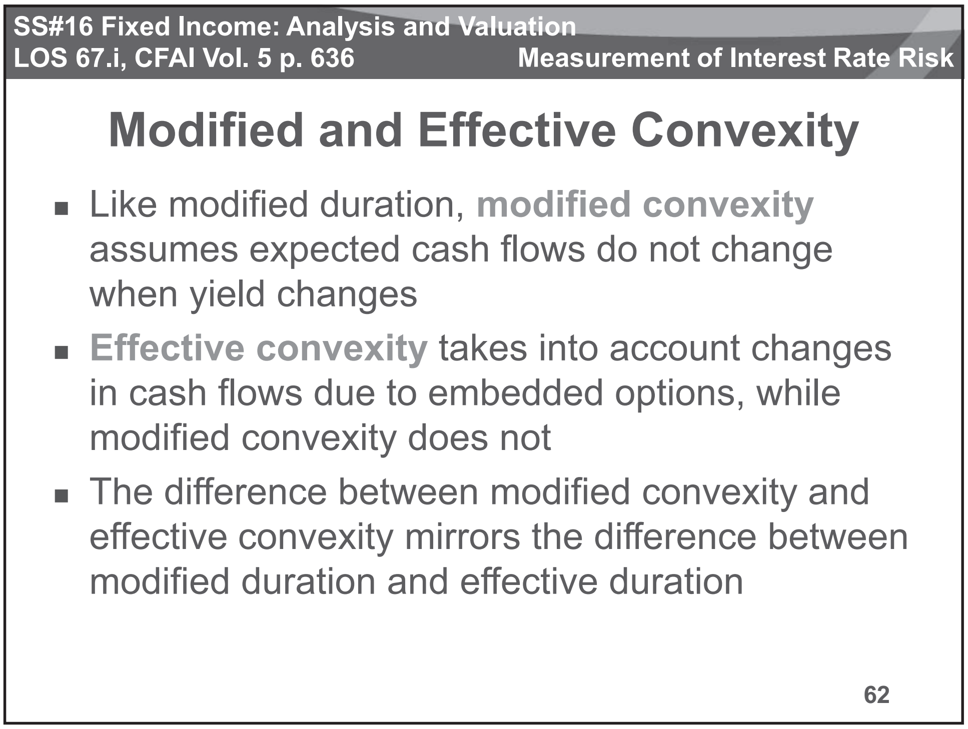
Duration only = +4.285% **Actual = +4.457%**

<u>For a yield increase of 0.5% we have</u>:

$-8.57\ (0.005) + 52.4\ (0.005)^2 = -4.154\%$

Duration only = –4.285% **Actual = –4.195%**

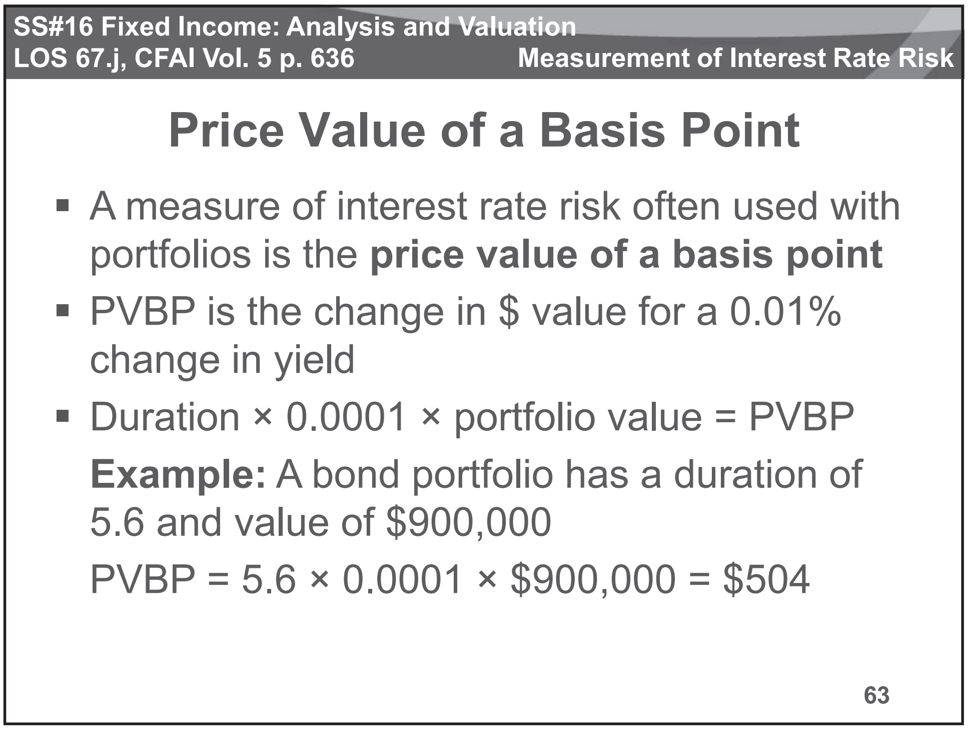
<u>Convexity adjustment improved both estimates</u>!



SS#16 Fixed Income: Analysis and Valuation
LOS 67.i, CFAI Vol. 5 p. 636 Measurement of Interest Rate Risk

Modified and Effective Convexity

- Like modified duration, **modified convexity** assumes expected cash flows do not change when yield changes
- **Effective convexity** takes into account changes in cash flows due to embedded options, while modified convexity does not
- The difference between modified convexity and effective convexity mirrors the difference between modified duration and effective duration



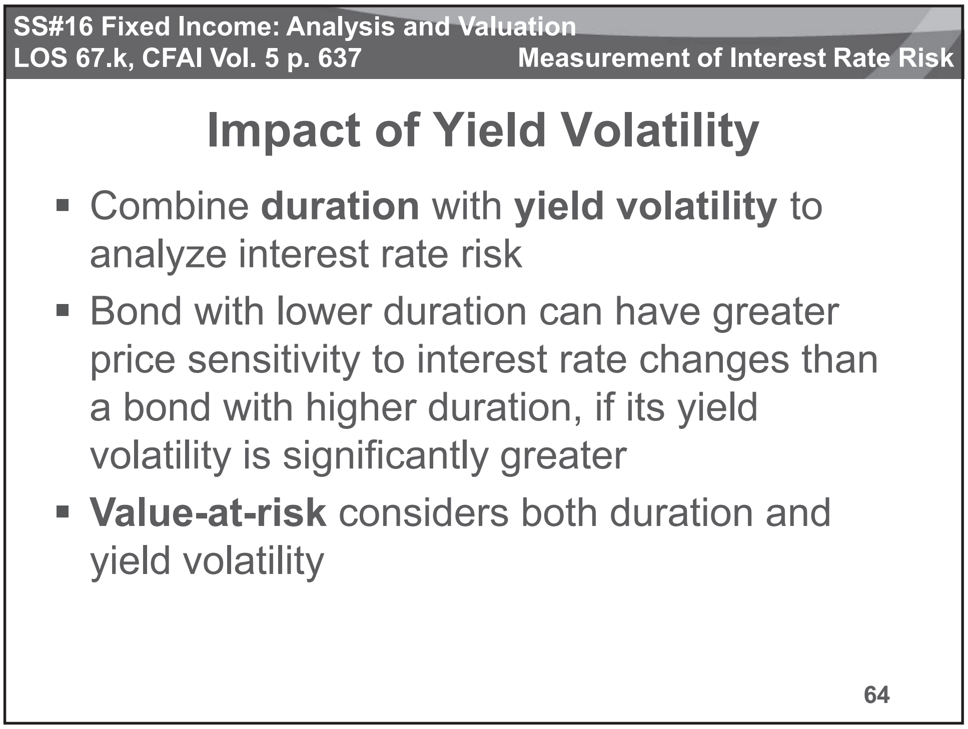
SS#16 Fixed Income: Analysis and Valuation
LOS 67.j, CFAI Vol. 5 p. 636 Measurement of Interest Rate Risk

Price Value of a Basis Point

- A measure of interest rate risk often used with portfolios is the **price value of a basis point**
- PVBP is the change in $ value for a 0.01% change in yield
- Duration × 0.0001 × portfolio value = PVBP

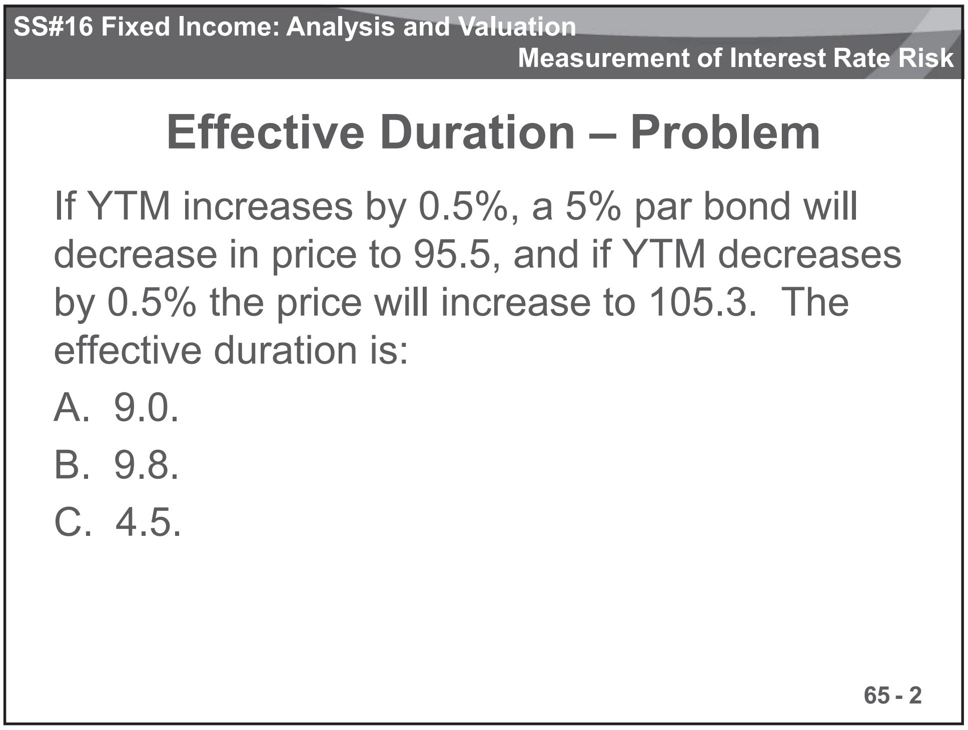
Example: A bond portfolio has a duration of 5.6 and value of $900,000

PVBP = 5.6 × 0.0001 × $900,000 = $504



Impact of Yield Volatility

- Combine **duration** with **yield volatility** to analyze interest rate risk
- Bond with lower duration can have greater price sensitivity to interest rate changes than a bond with higher duration, if its yield volatility is significantly greater
- **Value-at-risk** considers both duration and yield volatility



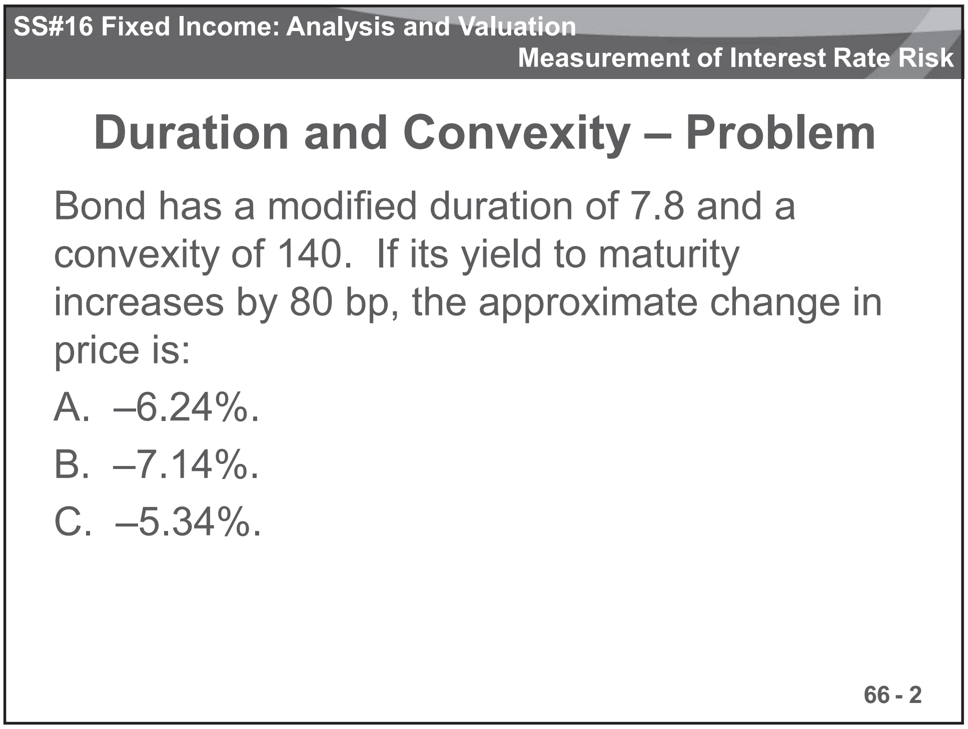
Effective Duration – Problem

If YTM increases by 0.5%, a 5% par bond will decrease in price to 95.5, and if YTM decreases by 0.5% the price will increase to 105.3. The effective duration is:

A. 9.0.

B. 9.8.

C. 4.5.



SS#16 Fixed Income: Analysis and Valuation

Measurement of Interest Rate Risk

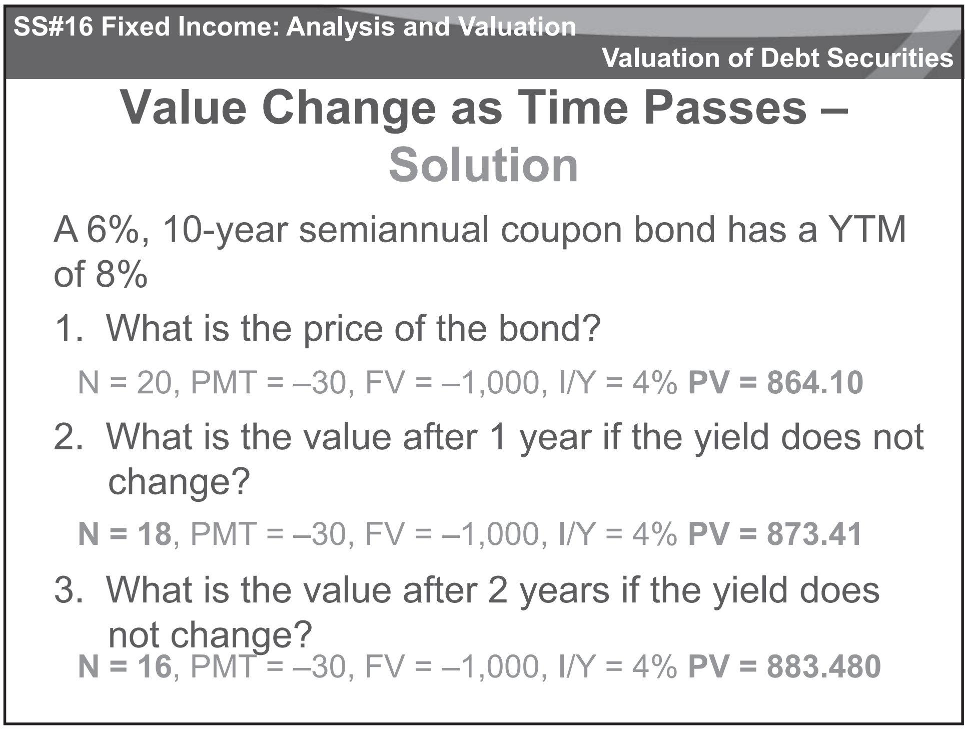
Duration and Convexity – Problem

Bond has a modified duration of 7.8 and a convexity of 140. If its yield to maturity increases by 80 bp, the approximate change in price is:

A. –6.24%.

B. –7.14%.

C. –5.34%.



SS#16 Fixed Income: Analysis and Valuation

Valuation of Debt Securities

Value Change as Time Passes – Solution

A 6%, 10-year semiannual coupon bond has a YTM of 8%

1. What is the price of the bond?

   N = 20, PMT = –30, FV = –1,000, I/Y = 4% **PV = 864.10**

2. What is the value after 1 year if the yield does not change?

   N = 18, PMT = –30, FV = –1,000, I/Y = 4% **PV = 873.41**

3. What is the value after 2 years if the yield does not change?

   N = 16, PMT = –30, FV = –1,000, I/Y = 4% **PV = 883.480**

SS#16 Fixed Income: Analysis and Valuation

Yield Measures, Spot Rates, Forward Rates

Equivalent Yields – Solution

An annual pay bond has a YTM of 14%.
The BEY for this bond is:

A. 13.54%.

$$2(\sqrt{1.14}-1)=0.1354$$

SS#16 Fixed Income: Analysis and Valuation

Yield Measures, Spot Rates, Forward Rates

Yield Measures – Solution

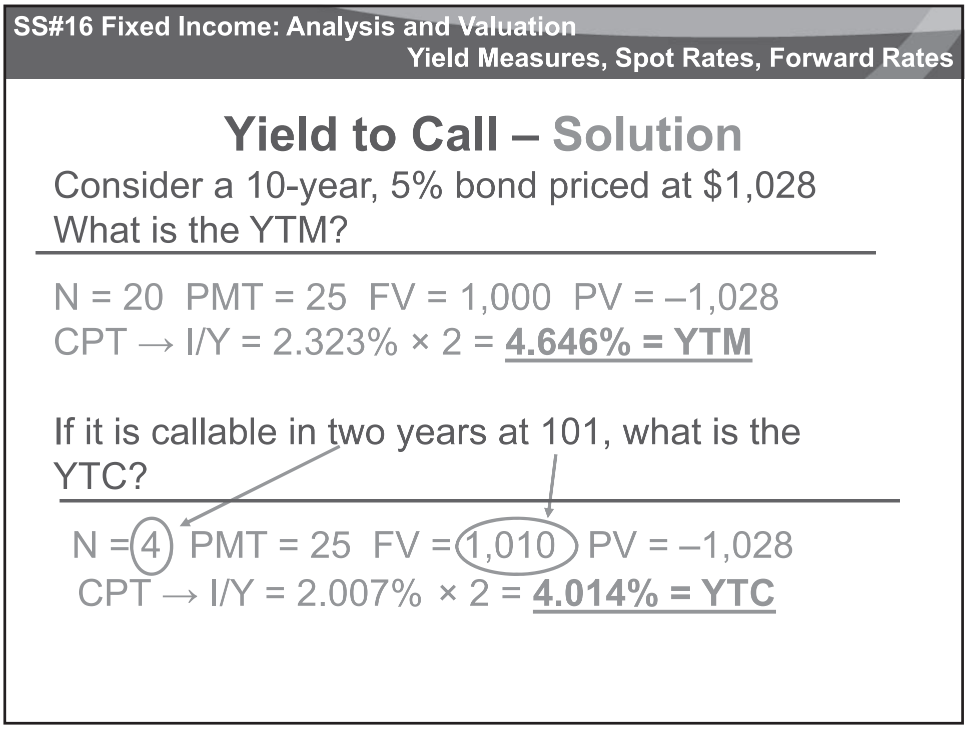
For a bond trading at a premium, order the coupon (nominal) yield, current yield, and YTM from smallest to largest.

$$\text{Current yield} = \frac{\text{Annual coupon}}{\text{Bond price}}$$

for premium bond, price > par

Current yield is less than coupon (nominal) yield

YTM is less than current yield for premium bond (movement towards par is negative)



SS#16 Fixed Income: Analysis and Valuation
Yield Measures, Spot Rates, Forward Rates

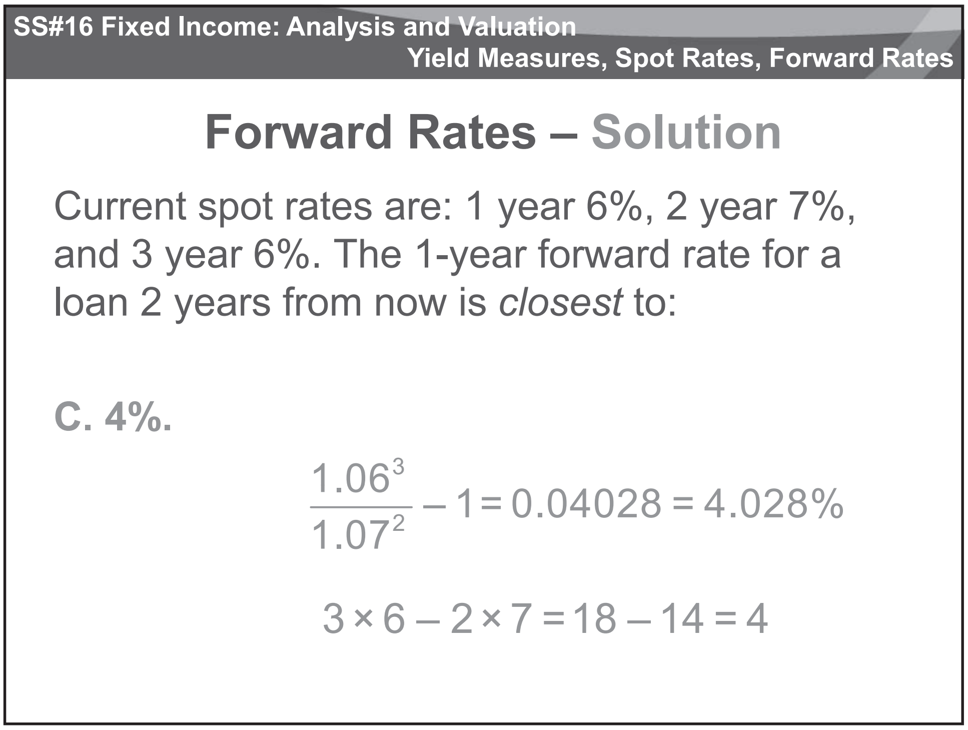
Yield to Call – Solution

Consider a 10-year, 5% bond priced at $1,028
What is the YTM?

N = 20 PMT = 25 FV = 1,000 PV = –1,028
CPT → I/Y = 2.323% × 2 = **4.646% = YTM**

If it is callable in two years at 101, what is the YTC?

N = $\boxed{4}$ PMT = 25 FV = $\boxed{1{,}010}$ PV = –1,028
CPT → I/Y = 2.007% × 2 = **4.014% = YTC**



SS#16 Fixed Income: Analysis and Valuation
Yield Measures, Spot Rates, Forward Rates

Forward Rates – Solution

Current spot rates are: 1 year 6%, 2 year 7%, and 3 year 6%. The 1-year forward rate for a loan 2 years from now is *closest* to:

C. 4%.

$$\frac{1.06^3}{1.07^2} - 1 = 0.04028 = 4.028\%$$

$$3 \times 6 - 2 \times 7 = 18 - 14 = 4$$

SS#16 Fixed Income: Analysis and Valuation

Measurement of Interest Rate Risk

Effective Duration – Solution

If YTM increases by 0.5%, a 5% par bond will decrease in price to 95.5, and if YTM decreases by 0.5% the price will increase to 105.3. The effective duration is:

B. 9.8.

$$\frac{105.3-95.5}{2(100)(0.005)}=9.8$$

SS#16 Fixed Income: Analysis and Valuation

Measurement of Interest Rate Risk

Duration and Convexity – Solution

Bond has a modified duration of 7.8 and a convexity of 140. If its yield to maturity increases by 80 bp, the approximate change in price is:

C. –5.34%.

$$-7.8(0.0080)+140(0.0080)^2=$$

$$-0.0624+0.00896=-5.344\%$$