VII. PERFORMANCE MEASUREMENTS

A. SIMPLE VERSUS COMPOUND INTEREST

1. Simple interest is the money earned by investing a sum of money for a specific period
   a. Interest = principal × interest rate × time

> **EXAMPLE** Johnny borrows $1,000 for 2 years at 8%. The total interest charged equals $160 ($1,000 × 2 × .08). Johnny would ultimately pay back $1,160 to the lender.

2. Simple interest is charged for borrowing money for a short period
3. Compound interest is basically interest earned on interest
   a. Mathematically, the growth is exponential (a power function) as opposed to linear (simple interest)
   b. An investment earns compound interest if accumulated interest is added to the principal (reinvested), so that future interest is earned both on the principal and any interest that has previously accumulated

> **EXAMPLE** Johnny invests $1,000 for 2 years at 8% compounded annually. How much will Johnny have in his investment after 2 years?
>
> PV = $1,000
>
> n = 2
>
> i = 8
>
> Solve for FV = $1,166.40

NOTE: Using compound interest generated an additional $6.40 when compared to the simple interest calculation used previously.

B. HOLDING PERIOD RETURN

1. The holding period return (also known as the single period return) is the basic method to evaluate the rate at which an investment grows or declines over time
2. The holding period return is determined by dividing the change in wealth by the initial investment
3. The formula for the holding period return is:

$$\text{HPR} = \frac{\text{ending value of investment} - \text{beginning value of investment} +/- \text{cash flows}}{\text{beginning value of investment}}$$

4. The holding period return is not often used as a measure of performance because it ignores the time value of money, and it does not address the time over which the investment has grown

EXAMPLE Bob purchases 100 shares of LAA stock for $40 per share. Two years later, Bob sells the 100 shares for $52 per share. In addition, Bob received a dividend of $2 per share in the first year and a dividend of $3 per share in the second year. His single period rate of return is calculated as follows:

Sales price (ending value) = $5,200 (100 × $52)

Purchase price (beginning value) = $4,000 (100 × $40)

Cash flow = $500 = (100 × $2) + (100 × $3)

Therefore, the holding period return is 42.5%

$$\frac{\$5{,}200 - \$4{,}000 + \$500}{\$4{,}000} = 42.5\%$$

EXAMPLE Harold purchases a Treasury bond for $1,050, receives interest of $160, and sells the bond for $980. His holding period return is 8.57%.

$$\frac{\$980 - \$1{,}050 + \$160}{\$1{,}050} = 8.57\%$$

C. ARITHMETIC MEAN

1. The arithmetic mean is calculated by dividing the sum of the holding period returns for each period by the total number of periods (n) being evaluated. The formula is written as follows:

$$AM = \frac{\sum_{t=1}^{n} HPR_t}{n}$$

EXAMPLE The arithmetic mean for the following set of data is 13.18%.

Year 1	Year 2	Year 3	Year 4	Year 5
15.2%	9.1%	6.5%	18.3	16.8%

$$AM = \frac{15.2\%+9.1\%+6.5\%+18.3\%+16.8}{5} = 13.18\%$$

2. The arithmetic mean is an approximation of the earnings rate for an investment over time
3. Large fluctuations in returns from year to year, especially negative returns, will have a tendency to cause the arithmetic mean to be misleading

D. GEOMETRIC MEAN

1. Geometric mean (GM) is the average compounded return or the internal rate of return (annualized return). The geometric mean is calculated using the following formula:

$$\text{geometric mean} = \sqrt[n]{(1+r_1)(1+r_2)(1+r_3)\ldots(1+r_n)} - 1$$

> **EXAMPLE** The geometric mean for the previous example is equal to 13.09%.
>
> $$GM = \sqrt[5]{(1+.152)(1+.091)(1+.065)(1+.183)(1+.168)} - 1$$
> $$GM = \sqrt[5]{1.8495} - 1$$
> $$GM = 13.09\%$$

2. The reason the arithmetic and geometric means differ is that the arithmetic mean does not consider the compounding effect of returns

> **EXAMPLE** If $1,000 is invested and earns 50% the first year and loses 50% the second year, the arithmetic mean equals 0% [(0.5 – 0.5) ÷ 2]. However, the geometric mean equals –13.39% $[(1 + 0.5) \times (1 + -0.5)]^{1/2} - 1$.
>
> This large discrepancy is because the first year yielded a 50%, or $500 return. The second year yielded a 50% loss; however, this loss was 50% of the $1,500, or $750. The arithmetic mean does not take this compounding effect into consideration and will, therefore, be subject to inaccuracy. Conceptually, the geometric mean is more accurate.

3. The geometric mean will always be less than the arithmetic mean, except when the returns for each period are equal, in which case the geometric and arithmetic means will be equal

4. The geometric mean is the same as the internal rate of return or annual compound return

> **EXAMPLE** If $1,000 is invested and earns the rates of returns previously illustrated, the IRR will be equal to the geometric mean. The first year 15.2% is earned, 9.1% in the second year, and so on. After the fifth year, the initial investment of $1,000 has grown to be $1,849.50. The IRR can be calculated as follows:
>
> PV = –1,000
>
> *n* = 5
>
> PMT = 0
>
> FV = 1,849.50
>
> Solve for *i*
>
> *i* = 13.09%
>
> Recall that the geometric mean also equals 13.09%.