The following is a review of the Debt Investments: Analysis and Valuation principles designed to address the learning outcome statements set forth by CFA Institute. This topic is also covered in:

Introduction to the Measurement of Interest Rate Risk

Study Session 16

Exam Focus

This topic review is all about the relation of yield changes and bond price changes, primarily based on the concepts of duration and convexity. There is really nothing in this study session that can be safely ignored; the calculation of duration, the use of duration, and the limitations of duration as a measure of bond price risk are all important. You should work to understand what convexity is and its relation to the interest rate risk of fixed income securities. There are two important formulas: the formula for effective duration and the formula for estimating the price effect of a yield change based on both duration and convexity. Finally, you should get comfortable with how and why the convexity of a bond is affected by the presence of embedded options. The LOS about the effect of a put option on the price-yield relationship is new this year.

LOS 1.C.a: Distinguish between the full valuation approach (the scenario analysis approach) and the duration/convexity approach for measuring interest rate risk, and explain the advantage of using the full valuation approach.

The **full valuation or scenario analysis approach** to measuring interest rate risk is based on applying the valuation techniques we have learned for a given change in the yield curve (i.e., for a given *interest rate scenario*). For a single option-free bond this could be simply, "if the YTM increases by 50 bp or 100 bp what is the impact on the value of the bond?" More complicated scenarios can be used as well, such as the effect on the bond value of a steepening of the yield curve where long term rates increase more than short term rates. If our valuation model is good, the exercise is straightforward: plug in the rates described in the interest rate scenario(s) and see what happens to the values of the bonds. If the valuation model used is sufficiently good, this is the theoretically preferred approach. Applied to a portfolio of bonds, one bond at a time, we can get a very good idea of how different changes in interest rates will affect the value of the portfolio.

The **duration/convexity approach** provides an approximation of the actual interest rate sensitivity of a bond or bond portfolio. Its main advantage is its simplicity compared to the full valuation approach. The full valuation approach can get quite complex and time consuming for a portfolio of more than a few bonds, especially if some of the bonds have more complex structures, such as call provisions. As we will see shortly, limiting our scenarios to parallel yield curve shifts and "settling" for an estimate of interest rate risk allows us to use the summary measures, duration and convexity. This greatly simplifies the process of estimating the value impact of overall changes in yield. Compared to the duration/convexity approach, the full valuation approach is more precise and can be used to evaluate the price effects of more complex interest rate scenarios than the duration-convexity approach, which strictly speaking is appropriate only for estimating the effects of parallel yield curve shifts.

LOS 1.C.b: Compute the interest rate risk exposure of a bond position or of a bond portfolio, given a change in interest rates.

In Figure 1, the full valuation approach is illustrated for Bond X, for Bond Y, and for a portfolio containing positions in both Bond X and Bond Y.

Consider two option-free bonds. Bond X is an 8 percent annual-pay bond with 5 years to maturity, priced at 108.4247 to yield 6 percent. (N = 5; PMT = 8.00; FV = 100; I/Y = 6.00%; CPT → PV = –108.4247).

Bond Y is a 5 percent annual-pay bond with 15 years to maturity, priced at 81.7842 to yield 7 percent.

Assume a $10 million face-value position in each bond and two scenarios. The first scenario is a parallel shift in the yield curve of +50 basis points and the second scenario is a parallel shift of +100 basis points. Note that the bond price of 108.4247 is the price per $100 of par value. With $10 million of par value bonds, the market value will be $10.84247 million.

Figure 1:

		Market Value of:			
Scenario	*Yield Δ*	*Bond X (in millions)*	*Bond Y (in millions)*	*Portfolio*	*Portfolio Value Δ%*
Current	+0 bp	$10.84247	$8.17842	$19.02089	–0.00%
1	+50 bp	$10.62335	$7.79322	$18.41657	–3.18%
2	+100 bp	$10.41002	$7.43216	$17.84218	–6.20%

N = 5; PMT = 8; FV = 100; I/Y = 6% + 0.5%; CPT → PV = –106.2335
N = 5; PMT = 8; FV = 100; I/Y = 6% + 1%; CPT → PV = –104.1002
N = 15; PMT = 5; FV = 100; I/Y = 7% + 0.5%; CPT → PV = –77.9322
N = 15; PMT = 5; FV = 100; I/Y = 7% + 1%; CPT → PV = –74.3216
Portfolio value change 50 bp: (18.41657 – 19.02089) / 19.02089 = –0.03177 = –3.18%
Portfolio value change 100 bp: (17.84218 – 19.02089) / 19.02089 = –0.06197 = –6.20%

It's worth noting that, on an individual bond basis, the effect of an increase in yield on the bonds' values is less for Bond X than for Bond Y (i.e., with a 50 bp increase in yield, the value of Bond X falls by 2.02 percent while the value of Bond Y falls by 4.71 percent; and with a 100 bp increase, X falls by 3.99 percent while Y drops by 9.12 percent). This, of course, is totally predictable since Bond Y is a longer-term bond and has a lower coupon—both of which mean more interest rate risk.

Professor's Note: Let's review the effects of bond characteristics on duration (price sensitivity). Holding other characteristics the same, we can state:

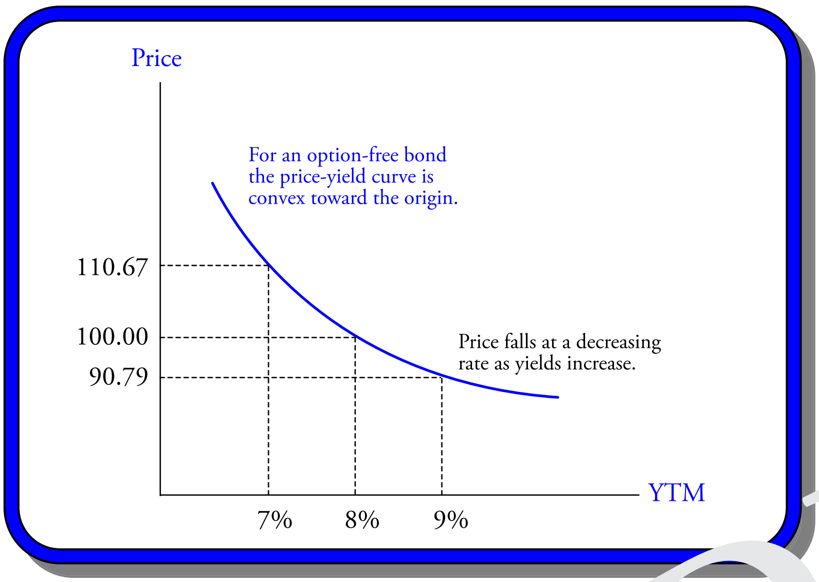
- *Higher (lower) coupon means lower (higher) duration.*
- *Longer (shorter) maturity means higher (lower) duration.*
- *Higher (lower) market yield means lower (higher) duration.*

Finance professors love to test these relations. (See Exam Flashbacks #1–#3.)

LOS 1.C.c: Demonstrate the price volatility characteristics for option-free bonds when interest rates change (including the concept of "positive convexity").

We established earlier that the relation between price and yield for a straight coupon bond is negative. An increase in yield (discount rate) leads to a decrease in the value of a bond. The precise nature of this relationship for an option-free, 8 percent, 20-year bond is illustrated in Figure 2.

Figure 2: Price-Yield Curve for an Option-Free, 8%, 20-Year Bond



First, note that the price yield relationship is negatively sloped, so the price falls as the yield rises. Second, note that the relation follows a curve, not a straight line. Since the curve is convex (toward the origin) we say that an option-free bond has positive convexity. Because of convexity, the price of an option-free bond *increases more when yields fall than it decreases when yields rise*. In Figure 2 we have illustrated that, for an 8 percent, 20-year option-free bond, a 1 percent decrease in the YTM will increase the price to 110.67, a *10.67 percent increase* in price. A 1 percent increase in YTM will cause the bond value to decrease to 90.79, a *9.22 percent decrease* in value.

If the price-yield relation were a straight line, there would be no difference between the price increase and the price decline in response to equal decreases and increases in yields. Convexity is a good thing for a bond owner; for a given volatility of yields, price increases are larger than price decreases. The convexity property is often expressed by saying, "a bond's price falls at a decreasing rate as yields rise." For the price-yield relationship to be convex, the slope (rate of decrease) of the curve must be decreasing as we move from left to right (i.e., towards higher yields).

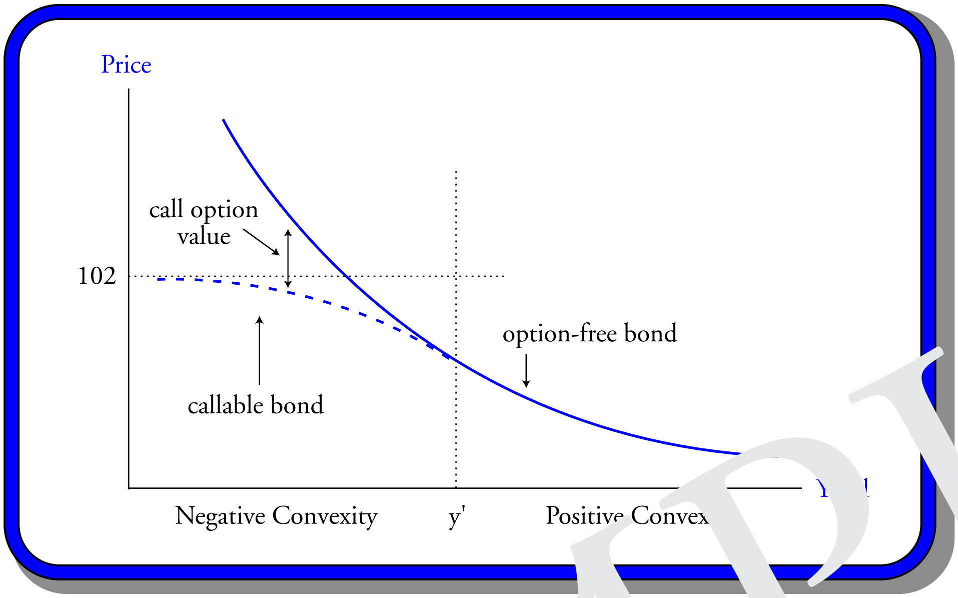
Note that the duration (interest rate sensitivity) of a bond at any yield is the (absolute value of) the slope of the price yield function at that yield. The convexity of the price yield relation for an option-free bond can help you remember a result presented earlier, that the duration of a bond is less at higher market yields. *(See Exam Flashback #4.)*

LOS 1.C.d: Demonstrate the price volatility characteristics of callable bonds and prepayable securities when interest rates change (including the concept of "negative convexity").

With a **callable or prepayable debt**, the upside price appreciation in response to decreasing yields is limited (sometimes called price compression). Consider the case of a bond that is currently callable at 102. The fact that the issuer can call the bond at any time for $1,020 per $1,000 of face value puts an effective upper limit on the value of the bond. As Figure 3 illustrates, as yields fall and the price approaches $1,020, the price-yield curve rises more slowly than that of an identical but noncallable bond. When the price begins to *rise at a decreasing rate* in response to further decreases in yield, the price yield curve "bends over" to the left and exhibits **negative convexity.**

Thus, in Figure 3, so long as yields remain *below level y'*, callable bonds will exhibit *negative convexity;* however, at yields *above level y'*, those same callable bonds will exhibit *positive convexity.* In other words, at higher yields the value of the call options becomes very small so that a callable bond will act very much like a noncallable bond. It is only at lower yields that the callable bond will exhibit negative convexity.

Figure 3: Price-Yield Function of a Callable Bond



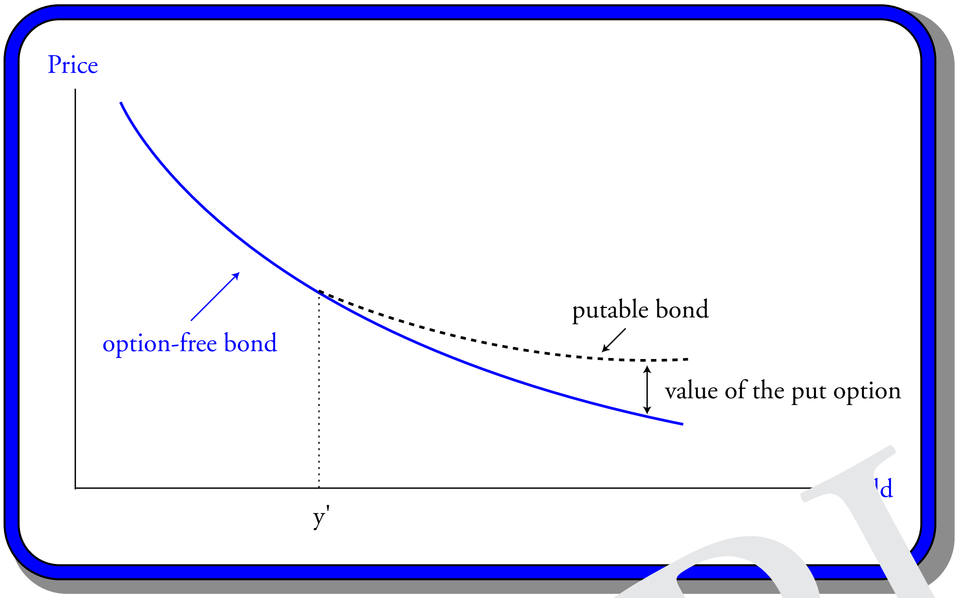
In terms of price sensitivity to interest rate changes, the slope of the price-yield curve at any particular yield tells the story. Note that as yields fall, the slope of the price-yield curve for the callable bond decreases, becoming almost zero (flat) at very low yields. This tells us how a call feature affects price sensitivity to changes in yield. At higher yields, the interest rate risk of a callable bond is very close or identical to that of a similar option-free bond. At lower yields, the price volatility of the callable bond will be much lower than that of an identical but noncallable bond.

The effect of a prepayment option is quite similar to that of a call; at low yields it will lead to negative convexity and reduce the price volatility (interest rate risk) of the security. Note that when yields are low and callable and prepayable securities exhibit less interest rate risk, reinvestment risk rises. At lower yields the probability of a call and the prepayment rate both rise, increasing the risk of having to reinvest principal repayments at the lower rates.

LOS 1.C: Describe the price volatility characteristics of putable bonds.

The value of a put increases at higher yields and decreases at lower yields opposite to the value of a call option. Compared to an option-free bond, a **putable bond** will have *less* price volatility at higher yields. This comparison is illustrated in Figure 4.

Figure 4: Comparing the Price-Yield Curves for Option-Free and Putable Bonds



In Figure 4, the price of the putable bond falls more slowly in response to increases in yield above *y*' because the value of the embedded put rises at higher yields. The slope of the price-yield relation is flatter, indicating less price sensitivity to yield changes (lower duration) for the putable bond at higher yields. At yields below *y*', the value of the put is quite small and a putable bond's price acts like that of an option-free bond in response to yield changes.

LOS 1.C.f: Compute the effective duration of a bond, given information about how the bond's price will increase and decrease for given changes in interest rates.

In our introduction to the concept of duration, we described it as the ratio of the percentage change in price to change in yield. Now that we understand convexity, we know that the price change in response to rising rates is smaller than the price change in response to falling rates for option-free bonds. The formula we will use for calculating the **effective duration** of a bond uses the average of the price changes in response to equal increases and decreases in yield to account for this fact. If we have a callable bond that is trading in the area of negative convexity, the price increase is smaller than the price decrease, but using the average still makes sense.

The formula for calculating the effective duration of a bond is:

$$\text{effective duration} = \frac{(\text{bond price when yields fall} - \text{bond price when yields rise})}{2 \times (\text{initial price}) \times (\text{change in yield in decimal form})}$$ which we will sometimes write as

$$\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)}$$

where:

V_- = bond value if the yield decreases by Δy

V_+ = bond value if the yield increases by Δy

V_0 = initial bond price

Δy = change yield used to get V_- and V_+, *expressed in decimal form*

Consider the following example of this calculation.

Example: Calculating effective duration

Consider a 20-year, semiannual-pay bond with an 8 percent coupon that is currently priced at \$908.00 to yield 9 percent. If the yield declines by 50 basis points (to 8.5 percent), the price will increase to \$952.30, and if the yield increases by 50 basis points (to 9.5 percent), the price will decline to \$866.80. Based on these price and yield changes, **calculate** the effective duration of this bond.

Answer:

Let's approach this intuitively to gain a better understanding of the formula. We begin by computing the average of the percentage change in the bond's price for the yield increase and the percentage change in price for a yield decrease. We can calculate this as:

$$\text{average } \textit{percentage} \text{ price change } = \frac{(952.30 - 866.80)}{2 \times 908.00} = 0.0471\%, \text{ or } 4.71\%$$

The "2" in the denominator is to obtain the average price change, and the 908 in the denominator is to obtain this average change as a percentage of the current price.

To get the duration (to scale our result for a 1 percent change in yield), the final step is to divide this average percentage price change by the change in interest rates that caused it. In the example, the yield change was 0.5 percent, which we need to write in decimal form as 0.005. Our estimate of the duration is:

$$\frac{0.0471}{0.005} = \frac{4.71\%}{0.50\%} = 9.42 = \text{duration}$$

Using the formula previously given, we have:

$$\text{effective duration} = \frac{(952.3 - 866.8)}{2 \times 908 \times 0.005} = 9.416$$

The interpretation of this result, as you should be convinced by now, is that a 1 percent change in yield produces an approximate change in the price of this bond of 9.42 percent. Note, however, that this estimate of duration was based on a change in yield of 0.5 percent and will perform best for yield changes close to this magnitude. Had we used a yield change of 0.25 percent or 1 percent, we would have obtained a slightly different estimate of effective duration.

This is an important concept and you are required to learn the formula for the calculation. To further help you understand this formula and remember it, consider the following.

The price increase in response to a 0.5% decrease in rates was $\frac{44.30}{908} = 4.879\%$. The price decrease in response to a 0.5% increase in rates was $\frac{41.20}{908} = 4.537\%$. The average of the percentage price increase and the percentage price decrease is 4.71 percent. Since we used a 0.5 percent change in yield to get the price changes, we need to double this and get a 9.42 percent change in price for a 1 percent change in yield. The duration is 9.42.

LOS 1.C.g: Compute the approximate percentage price change for a bond, given the bond's effective duration and a specified change in yield.

This is the same LOS we had in the previous study session. Multiply duration times the change in yield to get the magnitude of the price change and then change the sign to get the direction of the price change right (yield up, price down).

percentage change in bond price = –duration × change in yield in percent

Example: Using Duration

What is the expected percentage price change for a bond with an effective duration of 9 in response to an increase in yield of 30 basis points?

Answer:

$-9 \times 0.3\% = -2.7\%$

We expect the bond's price to decrease by 2.7 percent in response to the yield change. If the bond were priced at \$980, the new price is 980 × (1 – 0.027) = \$953.54. Don't make this hard; it's not.
(See Exam Flashback #5 and #6.)

LOS 1.C.h: Distinguish among modified duration, effective (or option-adjusted) duration, and Macaulay duration.

The formula we used to calculate duration based on price changes in response to equal increases and decreases in YTM, $\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)}$, is the formula for **effective (option-adjusted) duration.** This is the preferred measure because it gives a good approximation of interest rate sensitivity for both option-free bonds and *bonds with embedded options.*

Macaulay duration is an estimate of a bond's interest rate sensitivity based on the time, in years, until promised cash flows will arrive. Since a 5-year zero coupon bond has only one cash flow five years from today, its Macaulay duration is 5. The change in value in response to a 1 percent change in yield for a 5-year zero coupon bond is approximately 5 percent. A 5-year coupon bond has some cash flows that arrive earlier than five years from today (the coupons), so its Macaulay duration is less than 5. This is consistent with what we learned earlier: the higher the coupon, the less the price sensitivity (duration) of a bond.

Macaulay duration is the earliest measure of duration, and because it was based on the time, duration is often stated as years. Because Macaulay duration is based on the expected cash flows for an option-free bond, it is not an appropriate estimate of the price sensitivity of bonds with embedded options.

Modified duration is derived from Macaulay duration and offers a slight improvement over Macaulay duration in that it takes the current YTM into account. Like Macaulay duration, and for the same reasons, modified duration is not an appropriate measure of interest rate sensitivity for bonds with embedded options. For option-free bonds, however, effective duration (based on small changes in YTM) and modified duration will be very similar.

Professor's Note: The LOS here do not require that you calculate either Macaulay duration or modified duration, only effective duration. For your own understanding, however, note that the relation is:

Modified duration $= \frac{\text{Macaulay duration}}{1 + \text{periodic market yield}}$. *This accounts for the fact we learned earlier that duration decreases as YTM increases. Graphically, the slope of the price-yield curve is less steep at higher yields.*

LOS 1.C.i: Explain why effective duration, rather than modified duration or Macaulay duration, should be used to measure the interest rate risk for bonds with embedded options.

As noted earlier, in comparing the various duration measures, both Macaulay and modified duration are calculated directly from the promised cash flows for a bond with no adjustment for any embedded options. Effective duration is calculated from expected price changes in response to changes in yield that explicitly take into account a bond's option provisions (i.e., they are in the price-yield function used). *(See Exam Flashback #7.)*

LOS 1.C.j: Describe why duration is best interpreted as a measure of a bond's or portfolio's sensitivity to changes in interest rates.

We can interpret duration in three different ways.

First, duration is the slope of the price-yield curve at the bond's current YTM. Mathematically, the slope of the price-yield curve is the first derivative of the price-yield curve with respect to yield.

A second interpretation of duration, as originally developed by Macaulay, is a weighted average of the time (in years) until each cash flow will be received. The weights are the proportion of the total bond value that each cash flow represents. The answer, again, comes in years.

A third interpretation of duration is the approximate percentage change in price for a 1 percent change in yield. This interpretation, price sensitivity in response to a change in yield, is the preferred, and most intuitive, interpretation of duration.

Professors Note: The fact that duration was originally calculated and expressed in years has been a source of confusion for many candidates and finance students. Practitioners regularly speak of "longer duration securities." This confusion is the reason for this LOS. The most straightforward interpretation of duration is the one that we have used up to this point, "it is the approximate percentage change in a bond's price for a 1 percent change in YTM." I have seen duration expressed in years in CFA exam questions; just ignore the years and use the number. I have also seen questions asking whether duration becomes longer or shorter in response to a change; longer means higher or more interest rate sensitivity. A duration of 6.82 years means that for a 1 percent change in YTM, a bond's value will change approximately 6.82 percent. This is the best way to "interpret" duration.

LOS 1.C.k: Compute the duration of a portfolio, given the duration of the bonds comprising the portfolio.

The concept of duration can also be applied to portfolios. In fact, one of the benefits of duration as a measure of interest rate risk is that the duration of a portfolio is simply the weighted average of the durations of the individual securities in the portfolio. Mathematically, the duration of a portfolio is:

$$\text{portfolio duration} = w_1 D_1 + w_2 D_2 + \ldots + W_N D_N$$

where:

w_i = market value of bond i divided by the market value of the portfolio
D_i = the duration of bond i
N = the number of bonds in the portfolio

Example: Calculating portfolio duration

Suppose you have a two-security portfolio containing Bonds A and B. The market value of Bond A is $6,000, and the market value of Bond B is $4,000. The duration of Bond A is 8.5, and the duration of Bond B is 4.0. **Calculate** the duration of the portfolio.

Answer:

First, find the weights of each bond. Since the market value of the portfolio is \$10,000 = \$6,000 + \$4,000, the weight of each security is:

$$\text{weight in Bond A} = \frac{\$6{,}000}{\$10{,}000} = 60\%$$

$$\text{weight in Bond B} = \frac{\$4{,}000}{\$10{,}000} = 40\%$$

Using the formula for the duration of a portfolio, we get:

$$\text{portfolio duration} = (0.6 \times 8.5) + (0.4 \times 4.0) = 6.7$$

LOS 1.C.l: Explain the limitations of the portfolio duration measure.

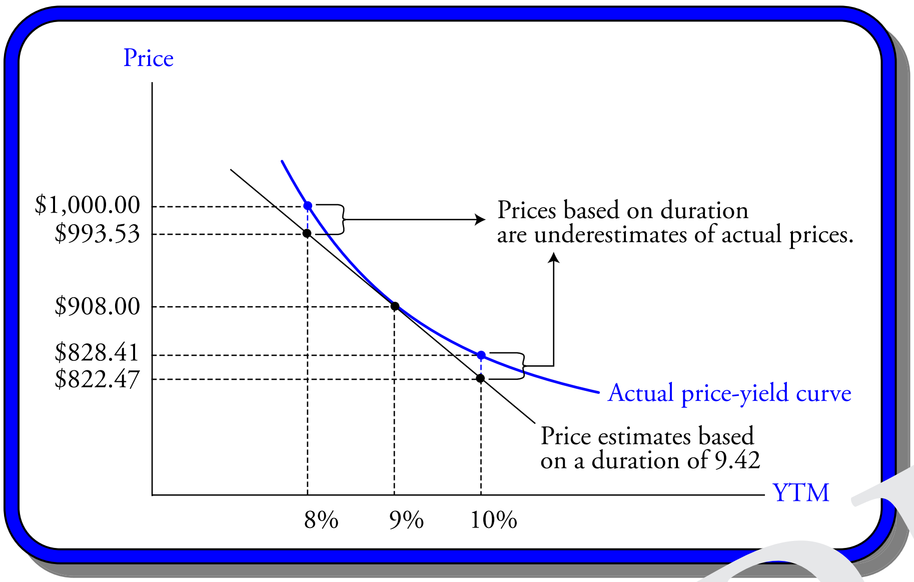
The limitations of portfolio duration as a measure of interest rate sensitivity stem from the fact that yields may not change equally on all the bonds in the portfolio. With a portfolio that includes bonds with different maturities, credit risks, and embedded options, there is no reason to suspect that the yields on individual bonds will change by equal amounts when the yield curve changes. As an example, a steepening of the yield curve can increase yields on long-term bonds and leave the yield on short-term bonds unchanged. It is for this reason that we say that duration is a good measure of the sensitivity of portfolio value to *parallel* changes in the yield curve.

LOS 1.C.m: Discuss the convexity measure of a bond.

Convexity is a measure of the curvature of the price-yield curve. The more curved the price-yield relation is, the greater the convexity. A straight line has a convexity of zero. If the price-yield 'curve' were, in fact, a straight line, the convexity would be zero. The reason we care about convexity is that the more curved the price-yield relation is, the worse our duration-based estimates of bond price changes in response to changes in yield are.

As an example, consider again an 8 percent, 20-year Treasury bond priced at \$908 so that it has a yield to maturity of 9 percent. We previously calculated the effective duration of this bond as 9.42. Figure 5 illustrates the difference between actual bond price changes and duration-based estimates of price changes at different yield levels.

Figure 5: Duration-Based Price Estimates vs. Actual Bond Prices



Based on a value of 9.42 for duration, we would estimate the price changes for 1% changes in yield (to 8% and 10%) as 1.0942 × 908 = $993.53 and (1 – 0.0942) × 908 = $822.47, respectively. These price estimates are shown in Figure 5 along the straight line tangent to the actual price-yield curve.

The actual price of the 8 percent bond at a YTM of 8 percent is, of course, par value ($1,000). Based on a YTM of 10 percent, the actual price of the bond is $828.41, about $6 higher than our duration based estimate of $822.47. Note that price estimates based on duration are less than the actual prices for both a 1 percent increase and a 1 percent decrease in yield.

Figure 5 illustrates why convexity is important and why estimates of price changes based solely on duration are inaccurate. If the price-yield relation were a straight line (i.e., if convexity were zero), duration alone would provide good estimates of bond price changes for changes in yield of any magnitude. The greater the convexity, the greater the error in price estimates based solely on duration. A method of incorporating convexity into our estimates of bond price changes in response to yield changes is the subject of the next LOS. *(See Exam Flashback #8.)*

LOS 1.C.n: Estimate a bond's percentage price change, given the bond's duration and convexity measure and a specified change in interest rates.

By combining duration and convexity we can obtain a more accurate estimate of the percentage change in price of a bond, especially for relatively large changes in yield. The formula for estimating a bond's percentage price change based on its convexity and duration is:

$$\text{percentage change in price} = \text{duration effect} + \text{convexity effect}$$

$$= \left\{\left[-\text{duration} \times (\Delta y)\right] + \left[\text{convexity} \times (\Delta y)^2\right]\right\} \times 100$$

With Δy entered as a decimal, the "× 100" is necessary to get an answer in percent.

Example: Estimating Price Changes with Duration and Convexity

Consider an 8 percent Treasury bond with a current price of \$908 and a YTM of 9 percent. **Calculate** the percentage change in price of both a 1 percent increase and a 1 percent decrease in YTM based on a duration of 9.42 and a convexity of 68.33.

Answer:

The duration effect, as we calculated earlier, is $9.42 \times 0.01 = 0.0942 = 9.42\%$. The convexity effect is $68.33 \times (0.01)^2 \times 100 = 0.00683 \times 100 = 0.683\%$. The total effect for a *decrease in yield of 1 percent* (from 9 percent to 8 percent) is: 9.42% + 0.683% = +10.103% and the estimate of the new price of the bond is 1.10103 × 908 = 999.74. This is much closer to the actual price of \$1,000 than our estimate using only duration.

The total effect for an *increase in yield of 1 percent* (from 9 percent to 10 percent) is: –9.42% + 0.683% = –8.737% and the estimate of the bond price is (1 – 0.08737)(908) = \$828.67. Again, this is much closer to the actual price (\$828.40) than the estimate based solely on duration.

There are a few points worth noting here. First, the convexity adjustment is always positive when convexity is positive because $(\Delta y)^2$ is always positive. This goes along with the illustration in Figure 5, which shows that the duration-only based estimate of a bond's price change suffers from being an underestimate of the percentage increase in the bond price when yields fell, and an overestimate of the percentage decrease in the bond price when yields rose. Recall, that for a callable bond, convexity can be negative at low yields. When convexity is negative, the convexity adjustment to the duration-only based estimate of the percentage price change will be negative for both yield increases and yield decreases.

Professor's Note: Different dealers may calculate the convexity measure differently. Often the measure is calculated in a way that requires us to divide the measure by two in order to get the correct convexity adjustment. For exam purposes, the formula we've shown here is the one you need to know. However, you should also know that there can be some variation in how different dealers calculate convexity. (See Exam Flashbacks #9 and #10.)

LOS 1.C.o: Differentiate between modified convexity and effective convexity.

Effective convexity takes into account changes in cash flows due to embedded options, while modified convexity does not. The difference between modified convexity and effective convexity mirrors the difference between modified duration and effective duration. Recall that modified duration is calculated without any adjustment to a bond's cash flows for embedded options. Also recall that effective duration was appropriate for bonds with embedded options because the inputs (prices) were calculated under the assumption that the cash flows could vary at different yields because of the embedded options in the securities. Clearly, effective convexity is the appropriate measure to use for bonds with embedded options, since it is based on bond values that incorporate the effect of embedded options on the bond's cash flows.

LOS 1.C.p: Compute the price value of a basis point (PVBP) and explain its relationship to duration.

The price value of a basis point (PVBP) is the dollar change in the price/value of a bond or a portfolio when the yield changes by one basis point or 0.01 percent. Using duration, we can calculate the price value of a basis point as:

duration × 0.0001 × bond value = price value of a basis point

The following example demonstrates this calculation.

Example: Calculating the Price Value of a Basis Point

A bond has a market value of $100,000 and a duration of 9.42. What is the price value of a basis point?

Answer:

Using the duration formula, the percentage change in the bond's price for a change in yield of 0.01 percent is: $0.01\% \times 9.42 = 0.0942\%$. We can calculate 0.0942 percent of the original $100,000 portfolio value as $0.000942 \times 100{,}000 = \$94.20$. If the bond's yield increases (decreases) by 1 basis point, the value will fall (rise) by $94.20. $94.20 is the (duration-based) price value of a basis point for this bond.

We could also directly calculate the price value of a basis point for this bond by increasing the YTM by 0.01 percent (0.0001) and calculating the change in bond value. This would give us the PVBP for an increase in yield. This would be very close to our duration-based estimate because duration is a very good estimate of interest rate risk for small changes in yield. We can ignore the convexity adjustment here because it is of very small magnitude: $(\Delta y)^2 = (0.0001)^2 = 0.00000001$, which is pretty small indeed!

KEY CONCEPTS

1. The full valuation approach to measuring interest rate risk involves using a pricing model to value individual bonds and can be used to find the price impact of any scenario of interest rate/yield curve changes. Its advantages are its flexibility and precision.
2. The duration/convexity approach is based on summary measures of interest rate risk and, while simpler to use for a portfolio of bonds than the full valuation approach, is theoretically correct only for parallel shifts of the yield curve.
3. Callable bonds and prepayable securities will have less interest rate risk (lower duration) at lower yields and putable bonds will have less interest rate risk at higher yields, compared to option-free bonds.
4. Option-free bonds have a price-yield relationship that is curved (convex toward the origin) and are, therefore, said to exhibit positive convexity. Bond prices fall less rapidly in response to yield increases than they rise in response to lower yields.
5. Callable bonds exhibit negative convexity at low yield levels; bond prices rise less rapidly in response to yield decreases than they fall in response to yield increases.
6. Effective duration is calculated as the ratio of the average percentage price change for equal increases and decreases in yield to the change in yield, effective duration $= \dfrac{V_- - V_+}{2V_0(\Delta y)}$.
7. Percentage change in bond price = –duration × change in yield in percent.
8. Macaulay duration and modified duration are based on a bond's promised cash flows, while effective duration takes into account the effect of embedded options on a bond's cash flows.
9. Effective duration is appropriate for estimating price changes in bonds with embedded options in response to yield changes; Macaulay and modified duration are not.
10. The most intuitive interpretation of duration is as the percentage change in a bond's price for a 1 percent change in yield.
11. The duration of a portfolio of bonds is equal to a weighted average of the individual bond durations, where the weights are the proportions of total portfolio value in each bond position.
12. Portfolio duration is limited in that it gives the sensitivity of bond portfolio value to yield changes that are equal for all bonds in the portfolio, an unlikely scenario for most portfolios.
13. Because of convexity, the duration measure is a poor approximation of price sensitivity for yield changes that are not absolutely small. The convexity adjustment accounts for the curvature of the price-yield relationship.

14. Incorporating both duration and convexity we can estimate the percentage change in price in response to a change in yield of (Δy) as: $\left\{\left[-\text{duration}\times(\Delta y)\right]+\left[\text{convexity}\times(\Delta y)^2\right]\right\}\times 100$.
15. Effective convexity considers expected changes in cash flows that may occur for bonds with embedded options, while modified convexity does not.
16. PVBP measures the price impact, in dollars, of a 1 basis point change in yield on a bond or bond portfolio.

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1

Source: Question #102 from '98 sample exam.

Bond price volatility normally is:
A. lower for higher coupons.
B. lower for longer durations.
C. greater for shorter maturities.
D. none of the above.

Exam Flashback # 2

Source: Question #85 from '01–'03 sample exams

A fixed income manager wants to take advantage of a forecast decline in interest rates over the next several months. Which of the following combinations of maturity and coupon rate would *most likely* result in the largest increase in portfolio value.

	Maturity	Coupon Rate
A.	2015	10%
B.	2015	12%
C.	2030	10%
D.	2030	12%

Exam Flashback # 3

Source: Question #50 from 93 actual exam.

Philip Morris has issued bonds that pay interest semiannually with the following characteristics:

Coupon	Yield-to-Maturity	Maturity	Modified Duration
8%	8%	15 years	10 years

Identify the direction of change in modified duration if the coupon of the bond were 4%, not 8%.
A. Increase.
B. Decrease.
C. No [illegible]
D. Cannot be determined with the information given.

Exam Flashback # 4

Source: Question #[illegible] from '91 actual exam.

Which one of the following is an *incorrect* statement concerning duration?
A. The higher the yield-to-maturity, the greater the duration.
B. The higher the coupon, the shorter the duration.
C. The difference in duration is small between 2 bonds each maturing in more than 15 years.
D. For a zero coupon bond, duration is the same, or very close to, the bond's term-to-maturity.

Exam Flashback # 5

Source: Question #106 from '92, '96 actual exams, and '97, '98 sample exams

A 9-year bond has a yield-to-maturity of 10% and a modified duration of 6.54 years. If the market yield changes by 50 basis points, the bond's expected price change is:
A. 3.27%.
B. 3.66%.
C. 5.00%.
D. 6.54%.

Exam Flashback # 6

Source: Question #48 from '91 actual exam.

An 8%, 15-year bond has a yield-to-maturity of 10% and a modified duration of 8.05 years. If the market yield changes by 25 basis points, how much of the change in the bond's price will be due to duration?
A. 1.85%.
B. 2.01%.
C. 3.27%.
D. 6.44%.

Exam Flashback # 7

Source: Question #100 from '99, '00, '01, '02, '03 sample exams.

Interest rate sensitivity for bonds with embedded options is *most accurately* measured by:
A. Convexity.
B. Effective duration.
C. Modified duration.
D. Macaulay duration.

Exam Flashback # 8

Source: Question #66 from '90, '91 actual exams.

Positive convexity on a bond implies that:
A. the direction of change in yield is directly related to the change in price.
B. prices increase at a faster rate as yields drop, than they decrease as yields rise.
C. price changes are the same for both increases and decreases in yields.
D. prices increase and decrease at a faster rate than the change in yield.

Exam Flashback # 9

Source: Question #9[illegible] from '94 actual exam.

A 6 percent coupon bond with semiannual coupons has a *convexity* of 60, sells for 80 percent of par, and is priced at a yield to maturity (YTM) of 9 percent. If the YTM increases to 9.5 percent, the predicted contribution to the *percentage* change in price, due to convexity, would be:
A. [illegible]
B. 1.35 %.
C. 2.48 %.
D. 7.35 %.

Exam Flashback # 10

Source: Question #44 from '91 actual exam.

A certain agency bond has a duration of 8.73 years and a convexity of 61.33. This implies that:
A. if market yields increase significantly (e.g., rates increase by 250 basis points), the price of the bond will fall by less than the amount indicated by duration alone.
B. if market yields increase significantly, the price of the bond will fall by more than the amount indicated by duration alone.
C. if market yields decrease significantly (e.g., by 250 basis points), the price of the bond will increase by less than the amount indicated by the convexity measure alone.
D. if market yields decrease significantly, the price of the bond will increase by less than the amount indicated by duration alone.

Concept Checkers: Introduction to the Measurement of Interest Rate Risk

1. Why is the price/yield profile of a callable bond less convex than that of an otherwise identical option-free bond? The price:
   A. increase is capped from above at or near the call price as the required yield decreases.
   B. increase is capped from above at or near the call price as the required yield increases.
   C. decrease is limited from below at or near the call price as the required yield decreases.
   D. decrease is limited from below at or near the call price as the required yield increases.

2. You own $15 million face of the 4.65 percent semiannual-pay Portage Health Authority bonds. The bonds have exactly 17 years to maturity and are currently priced to yield 4.39 percent. Using the full valuation approach, the interest rate exposure (in percent of value) for this bond position given a 75 basis point increase in required yield is *closest* to:
   A. –9.104%.
   B. –9.031%.
   C. –8.344%.
   D. –8.283%.

3. You are estimating the interest rate risk of a 14 percent semiannual-pay coupon with 6 years to maturity. The bond is currently trading at par. Using a 25 basis point change in yield, the effective duration of the bond is *closest* to:
   A. 0.389.
   B. 0.397.
   C. 3.889.
   D. 3.970.

4. Suppose that the bond in Question 3 is callable at par today. Using a 25 basis point change in yield, the bond's effective duration assuming that its price cannot exceed 100 is *closest* to:
   A. 1.972.
   B. 1.998.
   C. 19.72.
   D. [illegible]

5. Suppose that you determine that the modified duration of a bond is 7.87. The percentage change in price using duration for a yield decrease of 110 basis points is *closest* to:
   A. –8.657%.
   B. –7.155%.
   C. +7.155%.
   D. +8.657%.

6. A bond has a convexity of 57.3. The convexity effect if the yield decreases by 110 basis points is *closest* to:
   A. –1.673%.
   B. –0.693%.
   C. +0.693%.
   D. +1.673%.

7. Assume you're looking at a bond that has an effective duration of 10.5 and a convexity of 97.3. Using both of these measures, the estimated percentage change in price for this bond, in response to a decline in yield of 200 basis points is *closest* to:
   A. 22.95%.
   B. 19.05%.
   C. 17.11%.
   D. 24.89%.

8. An analyst has determined that if market yields rise by 100 basis points, a certain high-grade corporate bond will have a convexity effect of 1.75 percent. Further, she's found that the total estimated percentage change in price for this bond should be –13.35 percent. Given this information, it follows that the bond's percentage change in price due to duration is:
   A. –15.10%.
   B. –11.60%.
   C. +15.10%.
   D. +16.85%.

9. The total price volatility of a typical noncallable bond can be found by:
   A. adding the bond's convexity effect to its effective duration.
   B. adding the bond's negative convexity to its modified duration.
   C. subtracting the bond's negative convexity from its positive convexity.
   D. subtracting the bond's modified duration from its effective duration, then add any positive convexity.

10. The current price of a $1,000 7-year 5.5 percent semiannual coupon bond is $1,029.23 The bond's PVBP is *closest* to:
   A. $5.93.
   B. $0.60.
   C. $0.05.
   D. $5.74.

Answers – Exam Flashbacks

1. **A** Volatility and duration carry the same meaning in this question. High coupon bonds exhibit lower volatility/duration, holding other bond characteristics constant.

2. **C** You are looking for the bond that has the longer maturity and the lower coupon. Hence, Bond C most likely has the highest volatility level.

3. **A** As the coupon declines, the interest rate sensitivity of the bond (duration) increases.

4. **A** Because of convexity, duration is less for higher yields to maturity.

5. **A** $\Delta P/P = (-)(MD)(\Delta y) = (-)(6.54)(\pm 0.005) = \pm 0.0327$ or 3.27%

6. **B** $\Delta P/P = (-)(MD)(\Delta y) = (-)(8.05)(\pm 0.0025) = \pm 0.0201$ or 2.01%

7. **B** Effective duration accounts for changes in the curvature of the price-yield function that occur for bonds with embedded options. As yields fall, prices rise at a *decreasing rate* for bonds with embedded call options. Neither modified duration nor Macaulay duration would capture this impact. Convexity measures the rate of change in duration and is not a primary measure of interest rate sensitivity.

8. **B** Convexity is a measure of the rate of change in duration. Recall that duration is a measure of the slope of the price-yield function. Hence, as rates fall, the slope rises at an increasing rate and as yields rise the slope flattens out. Said differently, as rates fall, prices rise at an increasing rate and as rates rise, prices fall at a decreasing rate.

9. **B** $(C)(\Delta y)^2 = (60)(0.015)^2 = 0.0135 = 1.35\%$. Note that the decimal change in interest rates was used in this formula.

10. **A** Duration *overestimates* the decline in price due to an increase in interest rates. This is due to the fact that the price-yield function lies everywhere above the linear duration approximation.

Answers – Concept Checkers: Introduction to the Measurement of Interest Rate Risk

1. **A** As the required yield decreases on a callable bond, the rate of increase in the price of the bond begins to slow down and eventually level off as it approaches the call price, a characteristic known as "negative convexity."

2. **C** Price today = 103.092

$$N = 34;\ PMT = \frac{4.65}{2} = 2.325;\ FV = 100;\ I/Y = \frac{4.39}{2} = 2.195\%;\ CPT \rightarrow PV = -103.092$$

Price after 75 basis point increase in interest rates = 94.490

$$N = 34;\ PMT = \frac{4.65}{2} = 2.325;\ FV = 100;\ I/Y = \frac{5.14}{2} = 2.57\%;\ CPT \rightarrow PV = -94.490$$

$$\text{Interest rate exposure} = \frac{94.490 - 103.092}{103.092} = -8.344\%$$

3. **D** $V_- = 100.999$

$$N = 12;\ PMT = \frac{14.00}{2} = 7.00;\ FV = 100;\ I/Y = \frac{13.75}{2} = 6.875\%;\ CPT \rightarrow PV = -100.999$$

$V_+ = 99.014$

$$N = 12;\ PMT = \frac{14.00}{2} = 7.00;\ FV = 100;\ I/Y = \frac{14.25}{2} = 7.125\%;\ CPT \rightarrow PV = -99.014$$

$V_0 = 100.000$

$\Delta y = 0.0025$

$$\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)} = \frac{100.999 - 99.014}{2(100)0.0025} = 3.970$$

4. **A** $V_- = 100.000$

$V_+ = 99.014$

$V_0 = 100.000$

$\Delta y = 0.0025$

$$\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)} = \frac{100.000 - 99.014}{2(100)0.0025} = 1.972$$

5. **D** $\text{Est.}[\Delta V\_\%] =$ [illegible].87 × ([illegible]10%) = [illegible]57%

6. **C** convexity effect = convexity × $(\Delta y)^2 = \left[57.3(0.011)^2\right] \times 100 = 0.693\%$

7. **D** Total estimated price change = (duration effect + convexity effect)

$$\left\{\left[-10.5 \times (-0.02)\right] + \left[97.3 \times (-0.02)^2\right]\right\} \times 100 = 21.0\% + 3.89\% = 24.89\%$$

8. **A** Total percentage change in price = duration effect + convexity effect. Thus:

$$-13.35 = \text{duration effect} + 1.75 \Rightarrow \text{duration effect} = -15.10\%$$

(Note the duration effect must be negative because yields are rising.)

9. **A** Total percentage change in price = duration effect + convexity effect. Thus:

   Total percentage change in price = effective duration + convexity effect

   (Note that since this is a noncallable bond, you can use either effective or modified duration in the above equation.)

10. **B** PVBP = initial price – price if yield is changed by 1 bp. First, we need to calculate the yield so that we can calculate the price of the bond with a 1 basis point change in yield. Using a financial calculator: PV = –1,029.23; FV = 1,000; PMT = 27.5 = (0.055 × 1,000) / 2; N =14 = 2 × 7 years; CPT → I/Y = 2.49998, multiplied by 2 = 4.99995, or 5.00%. Next, compute the price of the bond at a yield of 5.00% + 0.01%, or 5.01%. Using the calculator: FV = 1,000; PMT = 27.5; N = 14; I/Y = 2.505 (5.01 / 2); CPT → PV = $1,028.63. Finally, PVBP = $1,029.23 – $1,028.63 = $0.60.